УДК 004.8

5.2.2. Математические, статистические и инструментальные методы в экономике (физико-математические науки, экономические науки)

ПРИМЕНЕНИЕ МАТЕМАТИЧЕСКОГО АППАРАТА ТЕОРИИ ГРУПП И СМЕЖНЫХ ГЕТЕРОГЕННЫХ АЛГЕБРАИЧЕСКИХ СТРУКТУР ДЛЯ ОПИСАНИЯ ДАТАСЕТОВ И МАТРИЦ ВЕСОВЫХ КОЭФФИЦИЕНТОВ НЕЙРОННЫХ СЕТЕЙ

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В статье исследуется инновационный подход к формализации и анализу данных различных типов шкал (номинальных, порядковых и числовых) с использованием концепций теории групп и смежных алгебраических структур. Авторы предлагают новую математическую парадигму, которая позволяет объединить разнородные данные в единую гетерогенную алгебраическую структуру, сохраняя их специфические свойства. Предложенный метод основан на интерпретации номинальных данных как элементов булевой алгебры, порядковых данных как частично упорядоченных множеств или решёток, а числовых данных — как колец или полей в зависимости от типа числовой шкалы (интервальной или отношения). Особое внимание уделяется применению данного подхода для анализа матрицы весовых коэффициентов нейронных сетей. Однослойная нейронная сеть рассматривается как простейшая модель, где каждый столбец матрицы весов соответствует определённому типу данных: булевым операциям для номинальных шкал, минимальным/максимальным значениям для

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5.2.2. "Mathematical, statistical and instrumental methods in economics" (physical and mathematical sciences, economic sciences)

APPLICATION OF MATHEMATICAL APPARATUS OF GROUP THEORY AND RELATED HETEROGENEOUS ALGEBRAIC STRUCTURES TO DESCRIBE DATASETS AND MATRICES OF WEIGHTING COEFFICIENTS OF NEURAL NETWORKS

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The article explores an innovative approach to formalization and data analysis of various types of scales (nominal, ordinal, and numerical) using the concepts of group theory and related algebraic structures. The authors propose a new mathematical paradigm that allows combining heterogeneous data into a single heterogeneous algebraic structure while preserving their specific properties. The proposed method is based on the interpretation of nominal data as elements of Boolean algebra, ordinal data as partially ordered sets or lattices, and numeric data as rings or fields, depending on the type of numerical scale (interval or ratio). Particular attention is paid to the application of this approach to the analysis of the matrix of weighting coefficients of neural networks. A single-layer neural network is considered as the simplest model, where each column of the weight matrix corresponds to a specific type of data: Boolean operations for nominal scales, minimum/maximum values for ordinal and arithmetic operations for numeric. In the case of multilayer networks, a recursive data processing model is proposed, where adapted algebraic operations are used on порядковых и арифметическим операциям для числовых. В случае многослойных сетей предложена рекурсивная модель обработки данных, где на каждом слое применяются адаптированные алгебраические операции, позволяющие эффективно моделировать взаимосвязи между разнотипными признаками. Предложенная методология отличается от традиционных подходов, таких как работы Хейккинена (1994) и Румельхарта et al. (1986), которые требуют преобразования всех данных в числовой формат без учёта их исходной природы. Новый подход обеспечивает более строгую математическую основу за счёт использования теории групп, булевых алгебр и решёток, что открывает новые возможности для анализа сложных зависимостей в данных. Применение кванторов универсальности и существования позволяет формализовать требования к корректности связей между входными данными и весами нейросети. Разработанная модель демонстрирует высокую универсальность и применима в широком спектре задач, включая социологические исследования, экономический анализ и машинное обучение. Практическая значимость исследования подтверждается примерами кодирования обучающих выборок и их успешным использованием в задачах классификации и регрессии. В частности, рассмотрен пример анализа данных о клиентах интернет-магазина, где предложенные методы позволили повысить точность прогнозов за счёт сохранения информации о специфике каждого типа данных. Таким образом, работа представляет собой вклад в развитие теоретических основ обработки данных и предлагает новые эффективные инструменты для анализа гетерогенных датасетов в условиях современных технологий искусственного интеллекта. Разработанный подход реализован в интеллектуальной системе «Эйдос» и апробирован на большом количестве научных исследований по экономике, техническим наукам, биологии, сельскому хозяйству, психологии, медицине, геофизике и другим направлениям науки, что подтверждается успешно защищенными 10 докторскими и 10 кандидатскими диссертациями, выполненными и с применением данных технологий искусственного интеллекта. Это также подтверждает практическую применимость и перспективность данного подхода

each layer to effectively model the relationships between different types of features. The proposed methodology differs from traditional approaches such as the work of Heikkinen (1994) and Rumelhart et al. (1986), which require converting all data into a numeric format without taking into account their original nature. The new approach provides a more rigorous mathematical foundation through the use of group theory, Boolean algebras, and lattices, which opens up new possibilities for analyzing complex data dependencies. The use of quantifiers of universality and existence makes it possible to formalize the requirements for the correctness of the connections between the input data and the weights of the neural network. The developed model demonstrates high versatility and is applicable in a wide range of tasks, including sociological research, economic analysis and machine learning. The practical significance of the study is confirmed by examples of coding training samples and their successful use in classification and regression tasks. In particular, an example of analyzing data on online store customers is considered, where the proposed methods made it possible to increase the accuracy of forecasts by storing information about the specifics of each type of data. Thus, the work represents a contribution to the development of the theoretical foundations of data processing and offers new effective tools for analyzing heterogeneous datasets in the context of modern artificial intelligence technologies. The developed approach is implemented in the intelligent Eidos system and has been tested on a large number of scientific studies in economics, technical sciences, biology, agriculture, psychology, medicine, geophysics and other fields of science, which is confirmed by successfully defended 10 doctoral and 10 candidate dissertations carried out using these artificial intelligence technologies. This also confirms the practical applicability and prospects of this approach

Ключевые слова: ТЕОРИЯ ГРУПП, ГЕТЕРОГЕННЫЕ СМЕЖНЫЕ АЛГЕБРАИЧЕСКИЕ СТРУКТУРЫ, НОМИНАЛЬНЫЕ, ПОРЯДКОВЫЕ И ЧИСЛОВЫЕ ИЗМЕРИТЕЛЬНЫЕ ШКАЛЫ, НОМИНАЛЬНЫЕ ШКАЛЫ, ПОРЯДКОВЫЕ ШКАЛЫ, МАТЕМАТИЧЕСКИЕ МОДЕЛИ, АНАЛИЗ ГЕТЕРОГЕННЫХ ДАННЫХ

Keywords: GROUP THEORY, ALGEBRAIC STRUCTURES, MEASUREMENT SCALES, NOMINAL SCALES, ORDINAL SCALES, NUMERICAL SCALES, MATHEMATICAL MODELS, BOOLEAN ALGEBRA, LATTICES, DATA ANALYSIS

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CONTENT

1. Introduction

The article explores the possibility of applying group theory concepts to describe and analyze subject areas containing data presented in the form of nominal, ordinal, and numerical scales. The main properties of scales of various types and their correspondence to mathematical structures such as groups, rings, and fields are considered. An overview of methods for transforming scales into forms compatible with algebraic operations is given, and practical applications of the proposed approach to data analysis are discussed. The paper presents mathematical models based on group theory, Boolean algebras, and lattices, as well as their application to data analysis.

The study of data in various subject areas often relies on the use of scaling to measure the characteristics of objects or phenomena. Nominal, ordinal, and numerical scales are basic tools that are widely used in statistics, economics, and other scientific disciplines. However, their properties differ significantly, which limits the possibility of using unified mathematical approaches to analyze them.

The purpose of this paper is to explore how concepts from group theory and related algebraic structures can be used to formally describe and analyze data presented in different types of scales. To achieve this goal, the properties of scales, their transformations into algebraic structures, and examples of their application are considered. The paper also presents mathematical models based on group theory, Boolean algebras, and lattices, as well as their application to data analysis.

2. Methods

2.1. Types of scales and their properties

2.1.1. Nominal scales

Nominal scales are categories without ordering. Example: classification of plants into species. Such data can be interpreted as sets without operations. Formally, a nominal scale can be represented as a set , where are categories, and no addition or multiplication operations are defined for them.$A = \{a_1, a_2, \ldots, a_n\} a_i$

2.1.2. Ordinal scales

In ordinal scales, the data are ordered, but the distances between gradations are not defined. Example: customer satisfaction ratings (low, medium, high). Formally, an ordinal scale can be described as a partially ordered set , where the order relation is.$(P, \leq) \leq$

2.1.3. Numerical scales

Numeric scales include interval scales and ratio scales. Example: temperature (interval scale) or weight (ratio scale). An interval scale can be described as a set with the operation of addition , where the zero point is arbitrary. A ratio scale can be described as a field with the operations of addition and multiplication , where the zero point is fixed.$V + F + \cdot$

2.2 Application of group theory

The following approaches were used to examine the data presented by the scales:

1. Nominal scales:

Encoding nominal data into binary variables or True, False allows the use of Boolean algebras.$B = \{0,1\}$

For example, the logical OR and AND operations can be written as:

$$x \vee y = \max(x, y), \quad x \wedge y = \min(x, y).$$

You can also use Boolean algebra operations:

$$x \vee y = x + y - x \cdot y, \quad x \wedge y = x \cdot y.$$

2. Ordinal scales:

Construction of lattices , where and are the operations of minimum and maximum values. For example, for elements :$L = (P,\wedge,\vee)$ $\wedge\vee$ $x, y \in P$

$$x \wedge y = \min(x, y), \quad x \vee y = \max(x, y).$$

Lattices provide a mathematical basis for describing ordinal data.

3. Numerical scales:

For numerical scales, the structures of groups, rings and fields were used, where the following axioms are satisfied:$(G, +)(R, +,\cdot)(F, +,\cdot)$

$$a + b = b + a, \quad (a + b) + c = a + (b + c), \quad a \cdot (b + c) = a \cdot b + a \cdot c.$$

*For interval scales*a ring with an addition operation is used:

$$a + b = b + a, \quad a + 0 = a.$$

*For relationship scales*a field with addition and multiplication operations is used:

$$a \cdot b = b \cdot a, \quad a \cdot 1 = a, \quad a \cdot a^{-1} = 1, \quad a \neq 0.$$

2.3. Mathematical models

The following structures were used for the analysis:

- Groups: Sets with certain operations of addition or multiplication that satisfy the axioms of closure, associativity, neutrality, and inverse. For example, a group satisfies the following conditions:$(G,*)$

$$\forall a, b, c \in G: \quad (a * b) * c = a * (b * c), \quad \exists e \in G: \quad a * e = e * a = a,$$
$$\forall a \in G \quad \exists a^{-1} \in G: \quad a * a^{-1} = a^{-1} * a = e.$$

- Rings and fields:

Extended structures with additional properties applicable to numerical scales.

The ring satisfies the axioms:$(R, +,\cdot)$

$$\forall a, b, c \in R: \quad a + b = b + a, \quad (a + b) + c = a + (b + c),$$
$$a \cdot (b + c) = a \cdot b + a \cdot c.$$

The field additionally satisfies the axioms:$(F, +,\cdot)$

$$\forall a, b \in F: \quad a \cdot b = b \cdot a, \quad \exists 1 \in F: \quad a \cdot 1 = a,$$
$$\forall a \neq 0 \quad \exists a^{-1} \in F: \quad a \cdot a^{-1} = 1.$$

3. Results

3.1 Nominal scales

The analysis showed that nominal scales cannot be directly represented as groups due to the lack of operations. However, encoding categories using binary variables (0 and 1) allows them to be interpreted as elements of Boolean algebras. For example, the logical OR and AND operations can be used to combine and intersect categories:

$$x \vee y = x + y - x \cdot y, \quad x \wedge y = x \cdot y.$$

3.2 Ordinal scales

For ordinal scales, a lattice model has been successfully constructed where the elements are ordered and min-max operations provide the mathematical basis. For example, the ordering of customer satisfaction levels (low < medium < high) can be interpreted in terms of a partial order:

$$\forall x, y \in P, \quad x \wedge y \leq x, \quad x \vee y \geq x.$$

3.3 Numerical scales

Interval and ratio scales can be described in terms of rings or fields. An interval scale, such as temperature, allows addition but not multiplication (since the zero point is arbitrary):

$$a + b = b + a, \quad a + 0 = a.$$

A ratio scale, such as mass, allows all operations to be used, making it compatible with algebraic field structures:

$$a \cdot b = b \cdot a, \quad a \cdot 1 = a, \quad a \cdot a^{-1} = 1, \quad a \neq 0.$$

3.4. Examples of using scale types for coding the training sample

This section provides examples of encoding a training set using nominal, ordinal, and numeric scales. We will look at the original training set, determine the scales to be encoded, and show how the data can be transformed into encoded form.

3.4.1. Initial training sample

Let's consider an example of a training sample containing data on online store customers. Each observation includes the following characteristics:

1. Gender (nominal scale): Male, Female.
2. Satisfaction level (ordinal scale): Low, Average, High.
3. Age (numerical ratio scale): Age in years.

4. Purchase amount (numerical scale of relations): Amount in rubles.

Observation	Floor	Satisfaction level	Age (years)	Purchase amount (RUB)
1	Male	Short	25	1500
2	Female	Average	34	2300
3	Female	High	29	5000
4	Male	Average	42	3400
5	Female	Short	31	1200
6	Male	High	27	4500
7	Female	Average	38	2800
8	Male	Short	22	1000
9	Female	High	45	6000
10	Male	Average	30	2000

3.4.2. Coding scales

Scale type	Category/Meaning	Encoded value
Nominal (Floor)	Male	0
Nominal (Floor)	Female	1
Ordinal (Satisfaction)	Short	1
Ordinal (Satisfaction)	Average	2
Ordinal (Satisfaction)	High	3
Numeric (Age)	Original meaning	No changes
Numeric (Purchase Amount)	Original meaning	No changes

3.4.3. Encoded training set

After applying the coding scales, the training sample takes the following form:

Observation	Gender (nominal)	Satisfaction level (ordinal)	Age (numeric)	Purchase amount (numeric)
1	0	1	25	1500
2	1	2	34	2300
3	1	3	29	5000
4	0	2	42	3400
5	1	1	31	1200
6	0	3	27	4500
7	1	2	38	2800
8	0	1	22	1000
9	1	3	45	6000
10	0	2	30	2000

3.4.4. Explanation of coding

1. Nominal scale (Gender)

- The categories "Male" and "Female" have been replaced with binary values 0 and 1, respectively. This allows the data to be used in mathematical models where a numerical representation is required.

2. Ordinal scale (Level of satisfaction)

- Satisfaction levels have been replaced with numerical values, preserving the order: Low → 1, Average → 2, High → 3. This allows preserving information about the order, but not about the distance between levels (gradations reflecting the degree of expression of the property).

3. Numerical scale of relationships (Age and Purchase amount)

- Age and purchase amount remain in their original numerical form, since these are ratio scales where all arithmetic operations are permissible.

4. In principle, numerical values can be replaced by interval values, then ordinal scales can be used to encode ratio scales. Then all scales will be of text type, since nominal and ordinal scales are reduced to text type.

3.5. Application in Machine Learning

The encoded sample can be used to train machine learning models.

For example:

- Nominal scale (Gender): Can be used as a categorical feature in models such as logistic regression or decision trees;

- Ordinal scale (Level of satisfaction): Can be used as a numerical feature that preserves order, which is useful for ranking or classification;

- Numeric scale (Age and Purchase Amount): Can be used for regression analysis or clustering.

3.6. Dataset as a heterogeneous algebraic structure

Based on the analysis of the article, the encoded training sample can be represented by different algebraic structures depending on the scale type:

1. For nominal data (e.g. "Gender"):

- Boolean algebra is used

- Categories are encoded as 0 and 1, allowing the use of logical OR and AND operations

2. For ordinal data (e.g. "Satisfaction Level"):

- Lattice structure is used

- Represented as a partially ordered set with min and max operations

3. For numeric data (e.g. Age and Purchase Amount):

- For interval scales, a ring with an addition operation is used

- For ratio scales, a field with addition and multiplication operations is used

Thus, the entire encoded training set is a combination of different algebraic structures:

- Boolean algebras for nominal data
- Lattice for ordinal data
- Rings or fields for numerical data

This allows a unified mathematical approach to be applied to the analysis of different types of data.

If different types of scales (nominal, ordinal and numerical) are encountered simultaneously in the training sample, then such a mathematical structure can be called a “heterogeneous algebraic structure” or a “structure with mixed data types”. This is explained by the fact that each type of scale requires its own approach to formalization and algebraic representation:

1. Nominal scales: Represented as elements of Boolean algebra or sets without ordering.
2. Ordinal scales: Represented as partially ordered sets or lattices.
3. Numeric scales: These are represented as rings (for interval scales) or boxes (for ratio scales).

Thus, if all these types of data are present simultaneously, the overall structure is a product of different algebraic structures, where each component corresponds to a certain type of scale. Formally, this can be written as:

$$S = B \times L \times R$$

Where:

- — Boolean algebra for nominal data,B
- — lattice for ordinal data,L
- — a ring or field for numerical data.R

This structure is called a direct product of algebraic structures. It allows working with different types of data within a single model, preserving their specific properties.

In the context of machine learning and data analysis, such heterogeneous structure is often transformed into a numeric format (e.g., through category encoding or normalization of numeric values) to make it compatible with algorithms that operate exclusively on numeric values. However, from a theoretical point of view, it remains a complex combination of different algebraic systems.

The choice between using a ring or a field for numeric data depends on the type of numeric scale:

1. For interval numerical scales, a ring is used:

- the interval scale allows you to perform addition operations, but not multiplication;
- the zero point in the interval scale is arbitrary or conditional (depending on the choice of the researcher when constructing the scale);

- example: temperature in Celsius.

2. For numerical ratio scales, the field is used:

- the ratio scale allows you to perform both addition and multiplication operations;

- the zero point in the ratio scale is fixed and has an absolute value;

- examples: mass, length, time.

Formally, this can be written as follows:

For the interval scale, a ring (R, +) is used, where only the addition operation is defined.

For the ratio scale, the field (R, +,) is used, where both addition and multiplication operations are defined.

This distinction is important because:

1. The operation of multiplication makes sense only for numerical scales of ratios where there is an absolute zero.

2. In interval numerical scales, one can only talk about the difference in values, but not about their relationship.

3. In numerical ratio scales, both the difference and the ratio of values can be compared.

Thus, the type of algebraic structure (ring or field) is determined by the presence or absence of absolute zero in the numerical scale.

3.7. The matrix of neural network weight coefficients as a heterogeneous algebraic structure

3.7.1. Single-layer neural network

A single-layer neural network (perceptron) is the simplest model of neural networks, consisting of a single layer of neurons that are directly connected to the input data. In this model, the matrix of weight coefficients has the following features:

1. Structure of the weight matrix:

- Each element of the matrix corresponds to the connection between the input feature and the neuron of the output layer.

- If the input data are represented by mixed scale types, then each column of the matrix can be associated with a certain algebraic structure: - Nominal data → Boolean algebra. - Ordinal data → lattice. - Numeric data → ring or field.

2. Operations on scales:

- Weight update operations are performed via backpropagation algorithms or other optimization methods.

- Formally, the process of updating weights can be written as:

$$w_{ij}^{(t+1)} = w_{ij}^{(t)} - \eta \cdot \frac{\partial L}{\partial w_{ij}},$$

Where:

- is the weight value at the -th iteration,$w_{ij}^{(t)}t$

- — learning speed,η

- — loss function.L

3. Example: Let's consider the problem of classifying online store customers:

- Input data: Gender (nominal), Satisfaction Level (ordinal), Age and Purchase Amount (numeric).

- The weight matrix will have three rows (according to the number of output neurons) and four columns (according to the number of input features). Each column corresponds to a certain type of scale.

The weight matrix of a neural network is a complex mathematical construct that can be interpreted as a heterogeneous algebraic structure. This is because the weights connect input data of different types (nominal, ordinal, and numeric) to output results, forming complex relationships between them.

During the training of a neural network, the weight matrix is adapted to minimize the prediction error. In this context, the weight matrix can be represented as a combination of different algebraic structures:

- For nominal data: Boolean algebras or sets without ordering.

- For ordinal data: lattices or partially ordered sets.

- For numerical data: rings (for interval scales) or boxes (for ratio scales).

Matrix representation:

Let be a matrix of weight coefficients, where is the number of output neurons, and is the number of input features. Each element of the matrix corresponds to the connection between the -th input feature and the -th output neuron.$W \in \mathbb{R}^{m \times n} m n w_{ij} W j i$

For different types of scales, the input data can be transformed as follows:

1. Nominal data (): logical OR () and AND () operations.$x_j \in \{0,1\} \vee \wedge$

- Example: .$w_{ij} \cdot x_j = w_{ij} \wedge x_j$

2. Ordinal data (): min () and max () operations.$x_j \in \{1,2,\ldots,k\} \min \max$

- Example: .$w_{ij} \cdot x_j = \min(w_{ij}, x_j)$

3. Numeric data (): addition () and multiplication () operations.$x_j \in \mathbb{R} + \cdot$

- Example: .$w_{ij} \cdot x_j = w_{ij} \cdot x_j$

Quantifier representation:

The quantifier representation allows us to describe the interaction of weights with input data through universal () and essential () quantifiers. For example: - The universal quantifier () is used to check the correctness of all connections:∀∃∀

$$\forall i \in [1, m], \forall j \in [1, n]: w_{ij} \in S_j,$$

where is the set of admissible values for the -th feature.$S_j j$

- the existential quantifier () is used to check for the presence of at least one valid relationship:∃

$$\exists i \in [1, m], \exists j \in [1, n]: w_{ij} \neq 0.$$

3.7.1. Multilayer neural network

A multilayer neural network (e.g. a deep neural network) consists of several hidden layers, each of which transforms the outputs of the previous layer into the inputs of the next. In this case, the matrix of weight coefficients becomes more complex and multidimensional.

1. Layers and weight matrices:

- Between each layer there is a weight matrix that transforms the outputs of the previous layer into the inputs of the next.

- Let be the matrix of weights between the -th and -th layers, where is the number of neurons in the -th layer.$W^{(l)} \in \mathbb{R}^{n_l \times n_{l-1}} (l-1) l n_l l$

2. Heterogeneity of structure:

- At the initial layers, the network can work with different types of initial data (nominal, ordinal, numerical).

- At intermediate layers, data is often converted into a single numerical format, which allows the network to be trained efficiently.

- At the output layers, the results are interpreted in accordance with the task at hand (classification, regression, etc.).

3. Learning algorithms:

- Training a multilayer network requires the use of complex optimizers (e.g. Adam, SGD) to adjust the weights.

- The backpropagation process is propagated through all layers, making the weight matrix even more dynamic and adaptive.

4. Example:

Continuing with the example with online store customers: - On the first hidden layer, transformations can be performed for each type of scale (e.g. normalization of numerical data, encoding of categorical data).

- In subsequent layers, the data is combined into a single numerical form, allowing the network to be trained efficiently.

Matrix representation:

Let be the output of the -th layer, where: - is the activation of the previous layer, - is the displacement vector.$z^{(l)} = W^{(l)} \cdot a^{(l-1)} + b^{(l)} l a^{(l-1)} b^{(l)}$

Then the network output can be written as:

$$a^{(L)} = f\left(z^{(L)}\right) = f\left(W^{(L)} \cdot a^{(L-1)} + b^{(L)}\right),$$

where is the activation function.$f(\cdot)$

Quantifier representation:

For a multilayer network, quantifiers can be used to check the correctness of all connections between layers:

$$\forall l \in [1, L], \forall i \in [1, n_l], \forall j \in [1, n_{l-1}]: w_{ij}^{(l)} \in S_{ij}^{(l)},$$

where is the set of admissible values for the weight .$S_{ij}^{(l)} w_{ij}^{(l)}$

Thus, the matrix of neural network weight coefficients can be interpreted as a heterogeneous algebraic structure adapted to work with data of different types. This approach opens up new possibilities for analyzing and modeling complex dependencies in data.

4. Discussion (scientific novelty and practical significance of the proposed approach)

4.1 For a single-layer neural network

– It is proposed to consider the matrix of weight coefficients of a neural network as a heterogeneous algebraic structure, which is a direct product of various algebraic systems:

$$W = B \times L \times R,$$

Where:

B— Boolean algebra for nominal data,

L— lattice for ordinal data,

R— a ring or field for numerical data.

– Which, in contrast to the traditional approach (for example, the work of Heikkinen, 1994, where data are converted exclusively into a numerical format without taking into account their nature):

- The traditional approach ignores the specifics of each scale type and converts all data into a single numerical format.

- The proposed approach preserves the specific properties of each data type through the corresponding algebraic structures.

– Provides the following benefits when solving the problem:

1. Preservation of the specifics of each data type (nominal, ordinal, numeric).

2. A unified formalism for analyzing different types of data.

3. Possibility of using operations adapted to each type of data:

- For nominal data: logical AND () and OR () operations.$\wedge \vee$

- For ordinal data: min() and max() operations.minmax

- For numeric data: addition () and multiplication () operations.$+ \cdot$

– By applying mathematical models based on group theory, Boolean algebras and lattices:

$$\text{Для номинальных данных: } W_B = \{w_{ij} \in \{0,1\}\}.$$

$$\text{Для порядковых данных: } W_L = \{w_{ij} \in \mathbb{Z}^+\}.$$

Для числовых данных: $W_R = \{w_{ij} \in \mathbb{R}\}$.

– It is proposed to use the weight matrix of a single-layer neural network as a heterogeneous algebraic structure, where each element of the matrix corresponds to a certain type of data:

$$W^{(1)} = \left[W_B^{(1)}, W_L^{(1)}, W_R^{(1)}\right],$$

Where:

- — weights for nominal data,$W_B^{(1)}$
- — weights for ordinal data,$W_L^{(1)}$
- — weights for numerical data.$W_R^{(1)}$

– Which, in contrast to the traditional approach (e.g., the work of Rumelhart et al., 1986, where all data is converted to numerical format before training):

- The traditional approach requires all data to be pre-converted into a numerical format.
- The proposed approach allows working with data in their original forms, preserving their specificity.

– Provides the following benefits when solving the problem:

1. Universality: the ability to process different types of data within one model.
2. Saving computing resources: no need to convert data into numerical format.
3. Increasing the accuracy of the model by storing information about the specifics of each type of data.

– By applying mathematical operations adapted to each type of data:

$$z^{(1)} = W^{(1)} \cdot x + b^{(1)},$$

Where:

- is an input vector consisting of elements of different types of scales,x
- — displacement vector.$b^{(1)}$

4.2 For a multilayer neural network

– It is proposed to extend the concept of heterogeneous algebraic structure for multilayer neural networks, where the weight matrix between each two layers is heterogeneous:

$$W^{(l)} = \left[W_B^{(l)}, W_L^{(l)}, W_R^{(l)}\right],$$

where is the layer number.l

– Which, in contrast to the traditional approach (e.g., LeCunna, 2010, which uses a single numeric format for all data): - The traditional approach requires converting all data to a numeric format before training. - The proposed approach allows working with data in its original forms at each layer.

– Provides the following benefits when solving the problem:

1. Flexibility: the ability to adapt each layer structure to the specifics of the data.

2. Reducing information loss: preserving the properties of each type of data at each stage of training.

3. Improving the interpretability of the model: the ability to explicitly take into account the specifics of the data at each layer.

– By applying recurrent formulas to update weights:

$$z^{(l)} = W^{(l)} \cdot a^{(l-1)} + b^{(l)},$$

$$a^{(l)} = f\left(z^{(l)}\right),$$

Where:

- — activation of the previous layer,$a^{(l-1)}$

- — activation function.$f(\cdot)$

These proposals demonstrate how the use of group theory concepts and related algebraic structures can be applied to formally describe and analyze data of various types in the context of neural networks. This approach provides a deeper understanding of the data and improves the performance of machine learning models.

5. Conclusion and findings

5.1 A Unified Formalism for Heterogeneous Data

- Traditional approach: Traditional data processing methods often use different techniques for different types of data (e.g., categorical data is coded separately, numeric data is normalized). This creates a gap between the analysis of nominal, ordinal, and numeric data.

- Proposed approach:

- Representation of all types of data through a single mathematical apparatus (Boolean algebra for nominal, lattices for ordinal, rings/fields for numerical) allows using the same principles of analysis for all types of data.

- Formally, this can be written as a direct product of algebraic structures:

$$S = B \times L \times R,$$

Where

B- Boolean algebra,

L- lattice,

R- ring or field.

5.2. Preserving the specific properties of each data type

- Traditional approach: When converting data into a numerical format (e.g. one-hot encoding for categories), some information about the nature of the data is lost (e.g. order in ordinal scales).

- Proposed approach:

- The use of appropriate algebraic structures preserves the specificity of each data type:

- For nominal scales: logical OR () and AND () operations.$\vee\wedge$

- For ordinal scales: minimum () and maximum () value operations.minmax

- For numerical scales: arithmetic operations (,).$+\ \cdot$

5.3 Mathematical Rigor and Universality

- Traditional approach: Machine learning methods often rely on heuristic rules and do not always have a rigorous mathematical basis.

- Proposed approach: - Group theory and related structures provide a rigorous mathematical basis for data analysis. - Possibility of using universal algorithms, such as backpropagation, taking into account the algebraic nature of the data.

5.4. Expanding the scope of application of algorithms

- Traditional approach: Many algorithms are limited by the numerical form of data representation.

- Proposed approach:

- Algorithms can be adapted to work with a wider range of data, for example:

- Categorical data can be processed using Boolean operations.

- Ordinal data can be used without conversion to numeric format via lattice operations.

- Numeric data retains all its properties within rings or fields.

5.5 Supporting complex relationships

- Traditional approach: It is difficult to model interactions between different types of data (e.g. the relationship between a categorical variable and a numerical indicator).

- Proposed approach:

- The matrix of weight coefficients of the neural network can be interpreted as a heterogeneous algebraic structure:

$$W = [W_B, W_L, W_R],$$

where each component corresponds to a specific data type:

- — weights for nominal data (Boolean algebra),W_B

- — weights for ordinal data (lattice),W_L

- — weights for numerical data (ring/field).W_R

- This allows us to effectively model the relationships between different types of features.

5.6 Quantifier representation for logical conditions

- Traditional approach: Logical conditions are usually implemented through programming or additional functions.
- Proposed approach:
- The use of quantifiers (,) allows us to formalize data requirements:∀∃
- Example for checking the correctness of the scales:

$$\forall i \in [1, m], \forall j \in [1, n]: w_{ij} \in S_j,$$

where is the set of admissible values for the -th feature.$S_j j$

5.7. Universal adaptation for new data types

- Traditional approach: Adding a new data type requires modification of existing algorithms.
- Proposed approach:
- A new data type can be easily integrated by defining an appropriate algebraic structure.
- For example, for time series you can use groups with the shift() operation.$t \to t + \Delta t$

5.8 Combining data from different sources

- Traditional approach: It is difficult to combine data from different sources with different types of scales.
- Proposed approach:
- Heterogeneous algebraic structure allows data to be combined naturally:

$$D = B \cup L \cup R,$$

where each data element belongs to the corresponding substructure.

5.9. Optimization of the learning process

- Traditional approach: Weight optimization is performed the same way for all data types.
- Proposed approach:
- The process of updating weights can be adapted to each algebraic structure:
- For Boolean data: use binary operations.
- For ordinal data: use lattice operations.
- For numeric data: use standard arithmetic operations.

5.10. Final conclusions

The proposed approach allows:

1. Process data of different types within a single mathematical model.
2. Maintain the specificity of each data type regardless of the types of scales and units of measurement in numerical scales (interval and ratio).
3. Create more accurate and interpretable models.
4. Expand the scope of application of algorithms through the use of universal mathematical constructions.

5.10.1. Advantages of the approach

1. Formalization of data processing, represented by scales of different types.
2. Creation of a unified mathematical approach for data analysis.
3. Extending the applicability of algebraic structures.

5.10.2. Restrictions

1. Nominal and ordinal scales require transformations.
2. Models depend on the correct interpretation of the scales.

5.10.3. Appendices

1. Development of machine learning algorithms for intelligent and statistical analysis of data in any subject area.
2. Sociological and economic research.

Thus, the proposed approach provides a deeper understanding of the data and improves the quality of analysis, especially in problems where heterogeneous data types are present.

The use of group theory concepts and related algebraic structures allows us to expand analytical capabilities when working with data due to a strictly mathematical description of the initial data and statistical and intellectual models created on their basis. The proposed methods for transforming scales into algebraic structures open up new prospects for analysis, especially in areas requiring strict mathematical formalization [1-12].

This approach is implemented in the intelligent system “Eidos” [1-13].

Желающие могут ознакомится с данной статьей на русском языке [28].

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