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4.3.1. Технологии, машины и оборудование для агропромышленного комплекса (технические науки, сельскохозяйственные науки)

РАСЧЕТ КОНЦЕНТРАЦИИ НАПРЯЖЕНИЙ ВБЛИЗИ ОСЕСИММЕТРИЧНОЙ ПОЛОСТИ В МАССИВЕ СОЛЯНЫХ ПОРОД МЕТОДОМ УПРУГОЙ АППРОКСИМАЦИИ

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Мы изучаем процесс аппроксимации последовательности, элементами которой являются упругие решения, полученные при изучении нелинейных задач теории вязкоупругости

Ключевые слова: СОЛЕВЫЕ ОТЛОЖЕНИЯ, ПОЛЗУЧЕСТЬ СОЛИ, ПОДЗЕМНЫЕ ВЫРАБОТКИ, ПОЛЯ ДЕФОРМАЦИЙ И НАПРЯЖЕНИЙ, МЕТОД УПРУГОЙ АППРОКСИМАЦИИ

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4.3.1. Technologies, machinery and equipment for the agro-industrial complex (technical sciences, agricultural sciences)

STRESS CONCENTRATION CALCULATION NEARBY AN AXISYMMETRIC CAVITY IN A SALT ROCK MASS BY THE ELASTIC APPROXIMATION METHOD

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We study the process of approximation of a sequence which elements are elastic solutions obtained in the study of nonlinear problems of the theory of viscoelasticity to the exact solution

Keywords: SALT SEQUENCES, SALT CREEP, UNDERGROUND WORKINGS, STRAIN AND STRESS FIELDS, ELASTIC APPROXIMATION METHOD

Mining workings are built in salt rocks, which are used for various purposes. It is known that at high values of rock pressure, salt rocks exhibit the properties of nonlinear creep. Creep of salts causes significant deformations, which can lead to loss of strength of structures erected by rocks.

An arbitrary choice of shapes, sizes of underground structures can cause their destruction, therefore, it is necessary to develop reasonable methods for calculating the stress concentration in the vicinity of underground structures based on rigorous mathematically sound models.

Currently, underground massifs of rock salts are used for the construction of cavities designed to store large volumes of gas, oil, oil products, as well as for the disposal of hazardous waste from various industrial productions. Such underground workings are widely used, since salt rocks are an ideal material for the construction of various storage facilities in them.

Salts are highly soluble in water, which allows the construction of cavities by an inexpensive method of deep erosion. In addition, the strength of salt rocks is such that it is possible to create large stable cavities in them. Salts are chemically inert in relation to oil products, and gases and various toxic wastes, therefore, they are impermeable to them, because. with high rock pressure, due to the creep property, all cracks in salt rocks are tightened and disappear. Compared to surface underground storage facilities for oil products and gases in salt products, they do not degrade the environment, do not occupy an area on the surface of the earth, and are safe from fire.

The choice of configuration and dimensions of the underground storage cavities depend on the deformation and strength characteristics of the salt layer, which has rheological properties. The construction and operation of stored in salt massifs is based on extensive practical experience, however, their strength calculations, except for some special cases, do not have sufficient scientific justification.

The well-known rigorous methods for calculating underground reservoirs make it possible to study only partial forms, that is, they do not provide a complete analysis of the strength of such storages.

The published approximate calculations of the stress distribution near storage cavities are based on salt deformation models that are not sufficiently experimentally substantiated and do not take into account the nonlinearity of the creep properties of the salt strata.

Storage tanks constructed underground are intended for long-term operation, therefore, in their strength calculations, it is necessary to rely on salt deformation models that take into account the effects of time in long-term experiments with uniaxial compression (tension) of the rod, and it is also necessary to conduct experiments on hollow samples of a cylindrical forms under load during the formation of a plane deformation.

The nonlinearity of the physical properties of rheological materials, taking into account the possibility of developing large deformations, requires the development of mathematical models that are designed to calculate stress and strain fields in structures and in their elements when their quasi-static equilibrium is studied.

The published materials, which offer the results of calculating the stress and strain fields that occur near the cavities, are based on linear elastic methods, and therefore are not accurate enough.

Analytical methods are used to study the stress concentration near the spherical and ellipsoidal shapes of storage tanks. These forms do not exhaust the many configurations of cavities built in salt pans.

In addition, the nonlinear properties of creep further complicate the methods for calculating the workings of salt deposits, so it is necessary to apply numerical methods for analyzing stress and strain fields that occur near storages of real configurations.

In particular, the finite element method is successfully used for such calculations. He found wide use in the calculations of various parts in rocket science, shipbuilding, and the study of the strength of platinum and slopes.

A more complete account of the mechanical properties of rock masses, in particular, salts, is possible using the finite element method. In order to improve the reliability and safety of underground storage cavities, it is necessary in the calculations to correctly set a quasi-static boundary value problem for an array with nonlinear viscoelastic properties and containing a cavity with axial symmetry using experimentally substantiated equations of the relationship between stresses and deformations of salt rocks.

When calculating the fields of deformations and stresses arising from the washing out of gas storage cavities in salt rocks, relations between stresses and deformations are described using the relations of the theory of nonlinear creep[3-6]

$$E\varepsilon_{ij}(t)=\sigma_{ij}(t)+\nu[\sigma_{ij}(t)-3\delta_{ij}\sigma(t)]+\frac{E}{2G}\int_0^t P[t,\tau,\sigma_{ij}(\tau)][\sigma_{ij}(\tau)-\delta_{ij}\sigma(\tau)]d\tau,$$

with core

$$P=D\tau^{-\alpha}\sigma_и^2(\tau),$$

in which$\sigma_{And}$is the stress intensity, andα, D – rheological constants of salt rock [3-6].

Let us consider the question of the convergence of a sequence whose elements are elastic solutions obtained in the study of nonlinear problems in the theory of viscoelasticity.

In [1], the dependences between stresses and strains of the nonlinear theory of viscoelasticity are mathematically substantiated.

In normed spaces $H_\varepsilon$and H_σ, whose elements are strain and stress tensors$\varepsilon_{ij}(\tau)$ And$\sigma_{ij}(\tau)$ respectively ($\tau\in[0,t]$), with norms

$$\|E\|=(\int_0^t\varepsilon_{ij}(\tau)\varepsilon_{ij}(\tau)\rho_\varepsilon(t,\tau)d\tau)^{0,5}\ ;\ \|\sigma\|=(\int_0^t\sigma_{ij}(\tau)\sigma_{ij}(\tau)\rho_\sigma(t,\tau)d\tau)^{0,5},$$

where functions$\rho_\varepsilon(t,\tau)>0,\rho_\sigma(t,\tau)>0$, it is shown in the monograph [1] that for any operators F, Q in mappingsσ=F(E), E=Q(σ) that are analytic in a neighborhood of zero, we have the representation

$$\begin{cases}F(E)=\sum\limits_{n=1}^{\infty}\overset{\vee}{\Gamma}_n E^n;\\ Q(\sigma)=\sum\limits_{n=1}^{\infty}\overset{\vee}{K}_n\sigma^n,\end{cases}$$

Where

$$\overset{\vee}{\Gamma}_n E^n = \int_0^t \ldots \int_0^t \Gamma_n(t,\tau_1,\ldots,\tau_n)\ E(\tau_1)\ldots E(\tau_n)d\tau_1\ldots d\tau_n;$$

$$\overset{\vee}{K}_n \sigma^n = \int_0^t \ldots \int_0^t K_n(t,\tau_1,\ldots,\tau_n)\ \sigma(\tau_1)\ldots\sigma(\tau_n)d\tau_1\ldots d\tau_n.$$

Let the salt deformation be described by the equations [4-6]

$$S_{ij}(t) = 2G\ell_{ij}(t) - 18DG^3\int_0^t \tau^{-\alpha}\ell_и^2(\tau)\ell_{ij}(\tau)d\tau, \qquad (1)$$

where functions

$$P(\ell_{ij}, S_{ij}) = \frac{1}{2G}S_{ij}(t) + 9DG^2\int_0^t K(\tau)\ell_и^2(\tau)\ell_{ij}(\tau)d\tau,$$

with 0≤τ≤α; 0≤t≤α; $\ell_и^2 = \frac{2}{3}\ell_{ij}\ell_{ij}$, and the kernel

$$K(\tau) = \begin{cases} \tau^{-\alpha}, & 0\le\tau\le t \\ 0, & \tau > t \end{cases}.$$

Let us assume that the deviatoric components of the stress and strain tensors Sij(τ), $\ell_{ij}(\tau)$ are elements of the space C [0,α], whose norm $\|h_{ij}\| = \max\limits_{0\le\tau\le\alpha} |h_{ij}|$.

Fréchet derivative of the operator P[(λ_{ij}(τ),Sij(τ)] has the form

$$P'_{\tilde{\ell}_{ij}}(\ell_{ij}) = 9DG^2\int_0^t \tau^{-\alpha} f[\tilde{\ell}_{ij}(\tau)]\ell_{ij}(\tau)d\tau,$$

Where

$$f(\tilde{\ell}_{ij}) = \begin{cases} \tilde{\ell}_и^2 + \frac{4}{3}\tilde{\ell}_{IJ}^2, & i = j \\ \tilde{\ell}_и^2 + \frac{8}{3}\tilde{\ell}_{IJ}^2, & i \ne j \end{cases}.$$

Let in the ball $R\left\|\ell_{ij}\right\| \le r$ function space $C[0,\alpha]$ the condition $\left\|P^{|}_{\tilde{\ell}_{ij}}(\ell_{ij})\right\| \le L$, then for arbitrary deviatoric components $\ell^{|}_{ij}$, $\ell^{||}_{ij} \in R$ the inequality [1]

$$\left\|P(\ell^{|}_{ij}, S_{ij}) - P(\ell^{||}_{ij}, S_{ij})\right\| \le L\left\|\ell^{|}_{ij} - \ell^{||}_{ij}\right\|.$$

It follows that if the parameter L is less than unity in the ball R with radius r, the operator P is contractive, and the method of successive approximations makes it possible to find its fixed point of the operator P. The value P[0,Sij(τ)], for which the inequality

$$\left\|P(0, S_{ij})\right\| \le (1-L)r. \qquad (2)$$

Using the estimate, we establish the condition for which the constant L<1:

$$\left\|P^{|}_{\tilde{\ell}_{ij}}(\ell_{ij})\right\| = \max_{0\le\tau\le\alpha}\left|9DG^2\int_0^t K(\tau)f[\tilde{\ell}_{ij}(\tau)]\ell_{ij}(\tau)\partial\tau\right| \le 9DG^2\left\|\ell_{ij}\right\| \cdot \left\|f(\tilde{\ell}_{ij})\right\| \cdot \frac{t^{1-\alpha}}{1-\alpha} \le 81DG^2r^3\frac{t^{1-\alpha}}{1-\alpha}.$$

As a result

$$L = 81DG^2r^3\frac{t^{1-\alpha}}{1-\alpha} < 1,$$

If

$$t < \left(\frac{1-\alpha}{81G^2Dr^3}\right)^{\frac{1}{1-\alpha}}. \qquad (3)$$

For inequality (2) to hold, we set

$$\left\|S_{ij}\right\| \le 2G(1-L)r, \qquad (4)$$

assuming that inequalities (3) and (4) hold.

Building two approximations with the application condition (1), we arrive at the relation [7-10]:

$$\ell_{ij}(t)=\frac{1}{2G}S_{ij}(t)+\frac{D}{2G}\int_0^t\tau^{-\alpha}\sigma_и^2(\tau)S_{ij}(\tau)d\tau, \qquad (5)$$

Where

$$\sigma_и^2=\frac{3}{2}S_{ij}S_{ij}.$$

In [1], the issue of convergence of a sequence of elastic solutions of problems in the theory of nonlinear creep is studied, when the physical and mechanical properties of the material have the form

$$S_{ij}(t)=\int_0^t\Gamma(t-\tau)\ell_{ij}(\tau)d\tau-\int_0^t\Gamma_\varphi(t-\tau)\varphi(\ell_и^2)\ell_{ij}(\tau)d\tau. \qquad (6)$$

Here

$$\Gamma(t)=\overset{0}{\Gamma}\delta(t)+\widehat{\Gamma}(t);\quad \Gamma_\varphi(t)=\overset{0}{\Gamma}_\varphi\delta(t)+\widehat{\Gamma}_\varphi(t).$$

The function u is the so-called generalized solution of the problem,

$$Sij,j(u)+\sigma_{,i}(u)+Fi=0 \qquad (7)$$

$$u|S=0, \qquad (8)$$

written in displacements if it is a solution to the equation

$$\int_v[S_{ij}(u)\ell_{ij}(w)+f_iw_i]=0,$$

in which the designations

$$f_i=\sigma_{,i}+F_i,\ \text{a}\ \sigma=\frac{1}{3}\sigma_{ii},$$

Let

.

In the space H with the norm

$$(u,w)=\int_\nu\ell_{ij}(u)\ell_{ij}(w)d\nu$$

(9)

by closing in the norm (9) the set of functions for which the conditions satisfying (7.8) are satisfied, a generalized solution is constructed.

Consider the equation

$$x = \int_0^t \Gamma(t-\tau)y(\tau)d\tau.$$

Let his only solution

$$y = \int_0^t K(t-\tau)x(\tau)d\tau,$$

and for $\varphi(\ell_и^2)$ And $\Gamma_\varphi(t)$ when t≥0 B the expressions are true

$$0 \le \varphi(\ell_и^2) \le \varphi(\ell_и^2) + \frac{\varphi(\ell_и^2) - \varphi(\ell_{и1}^2)}{\ell_и - \ell_{и1}} \ell_{и1} \le \eta,$$

(10)

$$|\Gamma_\varphi(t)| \le A\Gamma(t),$$

in which $A\eta = q < 1$.

Let the zero approximation be λ^{2I}, for which $\ell_{и0}^2 \le (1 - A\eta)M$, where M is some constant value, then we obtain the convergence of the sequence of elastic solutions of the problem of the theory of creep using (6).

Relations (1) are a consequence of (6) if we take in them

$$\overset{0}{\Gamma} = 2G, \quad \tilde{\Gamma}(t) = \overset{0}{\Gamma}_к = 0, \quad \varphi(\ell_и^2) = \ell_и^2,$$

$A\tilde{\Gamma}_\varphi(t-\tau)$ is given by the equality

$$\Gamma(\tau) = 18DG^3\tau^{-\alpha}.$$

The above theorem, which determines the conditions for the convergence of a sequence of elastic solutions, remains valid for (1), i.e., and for expression

(5).

If there is a condition

$$\max_{0\le\tau\le t} |\ell_{ij}| \le r,$$

For the quantityη=12r2 relation (10) is valid.

Ifηand zero approximation u0 correspond to the inequalities

$$\eta \le 10^{-3} \;\; ; \;\; \ell_{и}^{2}(u_0) \le (1-\eta)M, \qquad \text{(eleven)}$$

then the sequence of solutions of elastic problems for laws (1) and (5) will be convergent, because conditions (3), (4) and (11) for t∈[0–8.7·103 h] are performed.

To construct a convergent sequence of elastic solutions to the boundary value problem of the theory of creep, the elastic solution of this problem can be used as the initial approximation u0.

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