УДК 330.46 : 378.14

08.00.13 Математические и инструментальные методы экономики (экономические науки)

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ОПТИМАЛЬНОГО УПРАВЛЕНИЯ ПРОЦЕССОМ ОБУЧЕНИЯ

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В работе предлагается инновационный подход к распределению учебного времени между различными видами занятий студентов при заданном общем объеме нагрузки по определенной дисциплине. Он основан на разработанной автором математической модели оптимального распределения учебного времени между овладением знаниями и развитием умений. Модель описывается системой двух дифференциальных уравнений с управлением. С помощью принципа максимума Понтрягина найдено оптимальное управление для двух задач оптимизации. В первой из них требуется за минимальное время перейти из начальной точки фазовой плоскости в конечную. Установлено, что сначала следует достичь необходимого соотношения знаний и умений и выйти на магистраль, по которой и двигаться далее в течение основного периода учебного процесса. Показано, что при движении по магистрали распределение учебного времени между овладением знаниями и развитием умений - одно и то же для всех учащихся, а именно, треть времени следует отводить на лекции, две трети - на семинары. В конце основного периода следует сойти с магистрали и достичь заданной точки, либо только овладевая знаниями, либо только наращивая умения. Во второй задаче оптимизации необходимо возможно быстрее достичь заданного объема знаний, т.е. из исходной точки выйти на прямую на фазовой плоскости, определяемую заданным объемом знаний. Оптимальное управление описывается движением в течение трех периодов времени, первые два из которых - те же, что и при решении первой задачи. Сойти с магистрали следует так, чтобы половиной объема знаний студенты овладели в течение третьего периода, при этом умения не развивают. Полученные оптимальные траектории согласуются с опытом преподавания автором ряда дисциплин, в том числе организационно-экономического моделирования. Возможности практического

UDC 330.46 : 378.14

08.00.13 Mathematical and instrumental methods of Economics (Economic sciences)

MATHEMATICAL MODEL OF OPTIMAL MANAGEMENT OF THE LEARNING PROCESS

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The work proposes an innovative approach to the distribution of study time between different types of students' classes for a given total amount of workload in a particular discipline. It is based on the mathematical model developed by the author of the optimal distribution of study time between the acquisition of knowledge and the development of skills. The model is described by a system of two differential equations with control. With the help of Pontryagin's maximum principle, an optimal control is found for two optimization problems. In the first of them, it is required to pass from the initial point of the phase plane to the final one in the minimum time. It has been established that, first, it is necessary to achieve the necessary balance of knowledge and skills and enter the highway, along which to move further during the main period of the educational process. It is shown that when moving along the highway, the distribution of study time between mastering knowledge and developing skills is the same for all students, namely, one third of the time should be devoted to lectures, two thirds to seminars. At the end of the main period, one should leave the highway and reach a given point, either only by mastering knowledge, or only by increasing skills. In the second optimization problem, it is necessary to achieve a given amount of knowledge as quickly as possible, i.e. from the starting point to go to a straight line on the phase plane, determined by a given amount of knowledge. Optimal control is described by movement during three periods of time, the first two of which are the same as in the solution of the first problem. It is necessary to get off the highway in such a way that students master half of the amount of knowledge during the third period, while skills are not developed. The obtained optimal trajectories are consistent with the author's teaching experience in a number of disciplines, including organizational and economic modeling. The possibilities of practical application of the recommendations obtained on the basis of an innovative mathematical model deserve further применения полученных на основе инновационной математической модели рекомендаций заслуживают дальнейшего обсуждения

discussion

Ключевые слова: МАТЕМАТИЧЕСКИЕ МЕТОДЫ ЭКОНОМИКИ, МОДЕЛИРОВАНИЕ, УПРАВЛЕНИЕ, ИННОВАЦИИ, ОПТИМИЗАЦИЯ, ВЫСШЕЕ ОБРАЗОВАНИЕ, УЧЕБНЫЙ ПРОЦЕСС, ПРИНЦИП МАКСИМУМА ПОНТРЯГИНА

Keywords: MATHEMATICAL METHODS OF ECONOMY, MODELING, MANAGEMENT, INNOVATION, OPTIMIZATION, HIGHER EDUCATION, EDUCATIONAL PROCESS, PONTRYAGIN'S MAXIMUM PRINCIPLE

http://dx.doi.org/10.21515/1990-4665-185-006

Introduction

Management innovations are no less useful than innovations in the production of goods and services, but the economic effect of the introduction of management innovations is often difficult to express in monetary units. In accordance with the currently accepted classification of scientific specialties, management is one of the economic sciences. According to modern approaches to management, management decisions should be made on the basis of five groups of factors - social, technological, environmental, economic, political (for more details, see, for example, [1]). It is advisable to identify the effect of the introduction of managerial innovation within these five groups of factors, not limited to one of them - economic.

Management innovations include innovations in the field of education. Such innovations are manifold. For example, some of them relate to the content of education, others - to the systems of conducting classes (including the use of information and communication technologies), etc. One type of innovation in higher education is innovation in the organization of the educational process.

This paper proposes an innovative approach to the distribution of study time between different types of work of students for a given total amount of classes in a particular discipline. The recommendations obtained on the basis of the proposed approach are consistent with the teaching experience accumulated by the author and can be used by teachers of higher educational institutions in the preparation of curricula.

Building a mathematical model

The proposed innovative approach is based on a mathematical model of the optimal distribution of study time between the acquisition of knowledge and the development of skills. Let us consider the methodological prerequisites for constructing such a model.

Any knowledge consists partly of "information" ("pure knowledge") and partly of "skill" ("know how"). We will use the formulations of the famous mathematician and teacher D. Poya: "Skill is skill, it is the ability to use the information you have to achieve your goals; skill can also be described as a set of certain skills, in the end, skill is the ability to work methodically" [2, p.308].

It is inappropriate to engage in careful definitions of concepts that are well known to every teacher from practical teaching experience. We only note that the amount of "pure knowledge" of a student increases when listening to lectures and independent work, while the volume of his "skills" - while studying at seminars and practical classes, when doing laboratory work and homework, as well as when doing independent work. .

Practically important is the problem of the distribution of study time (classroom studies and independent work) between different types of student workload. In this case, the total amount of students' classes in a particular discipline is usually set. This assumption is natural, since the problem of distributing the total number of study hours between disciplines is usually solved at a higher level of managerial decision-making, namely, when drawing up a calendar plan for teaching students of a certain specialty.

We accept that the study time allocated for independent work of students is distributed between the increase in knowledge and the development of teachings in proportion to the division of classroom study time between these areas of activity. We will conditionally call "lectures" all classes aimed at increasing knowledge, and "seminars" - all classes aimed at developing skills. Then we can say that the problem considered in the article is to develop a mathematical apparatus for the optimal distribution of time between lectures and seminars.

Let us introduce the functions used in the proposed mathematical model. Let x(t) be the amount of information accumulated by the student by the time t (“pure knowledge”), y(t) – the amount of accumulated skills: the ability to reason, solve problems, understand the material presented by the teacher; u(t) is the share of time allotted for the accumulation of knowledge in the time interval (t; t + dt). Control is possible by choosing the best function u(t) from the point of view of one or another optimization problem.

The main thing in the model is the description of increments of knowledge and skills depending on the achieved values of these values.

Let us take as a starting point that the increment x(t + dt) – x(t) of the student's knowledge volume is proportional to the time spent on this u(t)dt and the accumulated skills y(t). In other words,

$$\frac{dx(t)}{dt} = k_1 u(t) y(t) .(1)$$

In formula (1), the coefficient k1 > 0 is determined by the individual characteristics of the student in question.

The second starting position is that the increment of skills y(t + dt) - y(t) over time from t to t + dt is proportional to the time spent on this (1 - u(t))dt, available skills y(t) and knowledge x(t). Consequently,

$$\frac{dy(t)}{dt} = k_2 (1 - u(t)) x(t) y(t) . (2)$$

The positive coefficient k2 in formula (2), as well as in formula (1), is determined by the individual characteristics of the student.

Let us explain the starting points. The model assumes that the student acquires skills the faster, the more he already knows and can do (formula (2). At the same time, he learns knowledge the faster, the more he can, regardless of previously accumulated knowledge (formula (1)) In our opinion, it cannot be assumed that the more a student remembers, the faster he remembers new information.This explains the fact that the right side of equation (1) is affected only by those knowledge acquired in the past that became active, since they were applied in solving tasks and, as a result, turned into skills.

Note that the model (1) - (2) makes sense to apply on such time intervals that, for example, an academic hour can be considered an infinitesimal value. Instead of the system of differential equations (1) - (2), one could consider a system of difference equations, however, from a mathematical point of view, it is preferable to analyze the system of differential equations, since a well-developed theory of optimal control can be applied in this case.

To study and use the model (1) - (2) in order to organize the educational process, there is no need to develop specific methods for assessing the functions used - skills y(t) and knowledge x(t). It suffices to assume that these functions exist and satisfy equations (1) - (2). The model makes it possible to catch the main interrelations of the variables used and obtain practically useful conclusions without going into details of finding (evaluating) skills y(t) and knowledge x(t), since the main thing in it is finding the optimal control over the distribution of study time, i.e. functions u(t). Models of this type V.V. Nalimov calls sketches [3], since they are aimed at identifying the interrelations of variables of interest to the researcher without working out the issues of measuring these variables.

We can control the learning process by choosing for each t the value of the function u(t) from the segment [0; one]. The following two optimization problems are useful for planning the learning process.

1. How to achieve a given level of knowledge x1 and skills y1 as quickly as possible? In other words, how to move in the shortest possible time from the starting point of the phase plane (x0; y0), which reflects the level of knowledge

and skills of the student before the start of training, to the target point (x1; y1) set by the organizers of the educational process?

2. How to act in order to achieve the given amount of knowledge as quickly as possible, i.e. from the starting point (x0; y0), go to the line x = x1?

The results of solving the dual problem are also useful for practice: to achieve as much knowledge as possible in a given time. The optimal motion trajectories for the second problem and its dual coincide (duality is understood in the usual sense for mathematical programming - see, for example, [4]).

It turns out that the system of equations (1) - (2) can be simplified. Having made the change of variables z = k2x, w = k1k2y, we pass from (1) – (2) to a simpler system of differential equations, in which there are no unknown coefficients:

$$\frac{dz}{dt} = uw, \quad \frac{dw}{dt} = (1-u)zw \ . \ (3)$$

The above linear change of variables means the transition to other units of measurement of knowledge and skills, while each student uses his own personal system of units of measurement, determined by his personal properties, which in equations (1) - (2) were reflected by the coefficients k1 and k2. Thus, system (1) - (2) describes the learning process of all students, the volumes of knowledge and skills are measured uniformly (and, in principle, it is possible to develop objective rules for assessing these volumes based on the content of the discipline being studied), while the individual characteristics of students are taken into account coefficients k1 and k2, and system (3) does not contain unknown coefficients, and therefore its solution describes the dynamics of measurements of knowledge volumes and skills of each student. However, the units of measurement of these volumes are different for each of them.

Type of optimal control

Let's start by studying system (3). To solve the above problems 1 and 2, we apply mathematical methods of optimal control. The best form of the time distribution control function u(t) can be found using the maximum principle of L.S. Pontryagin [5, 6]. A detailed description of the process of solving mathematical problems 1 and 2 is not the subject of this work. We only note that there are no fundamental differences from solving other optimal control problems by applying the maximum principle of L.S. Pontryagin.

We present the results obtained. Let's start with problem 1 for system (3). From the maximum principle of L.S. Pontryagin it follows that the fastest motion can occur either along horizontal (for them u = 1) or vertical (for them u = 0) line segments, or according to a special solution, which is the parabola w = z2 (u = 1/3). For starting points below the parabola, i.e. at$z_0^2 > w_0$, the movement should begin along a segment of a vertical straight line. For starting points above the parabola, i.e. at$z_0^2 < w_0$, the movement first goes along a segment of a horizontal straight line. If the starting point lies on the parabola, i.e.$z_0^2 = w_0$, then the fastest motion occurs along this parabola. It is important that for each of the regions below and above the parabola, i.e. {z2 > w} and {z2 < w}, respectively, can pass at most one vertical and one horizontal segment of the optimal trajectory.

Based on the regular synthesis theorem [7, p.266], the optimal trajectory is found. It looks like this. First you need to go to the "highway" - get to the parabola w = z2 along a vertical (if the starting point is below the parabola) or horizontal (if the starting point is above the parabola) straight line segment. Then the main part of the path should be passed along the highway, which in the problem under consideration is a parabola (for it u = 1/3). If the end point lies under the parabola, you need to get to it horizontally, leaving the highway. If it lies above the parabola, then the final section of the trajectory is a vertical segment. In the case when the end point lies on a parabola, you should stop at it after moving along the highway.

For example, in the case$w < z_0^2 < w < z_1^2$and initial(z_0, w), and the end point(z_1, w)lie under the parabola. Therefore, the optimal trajectory is as follows. First you need to go to the highway - get along the vertical (u = 0) straight line to the parabola, i.e. go from point(z_0, w)exactly(z_0, z_0^2). Then you should move along the highway (u = 1/3) from the point$(z_0; z_0^2)$to the point$(\sqrt{w_1}; w_1)$. After that, you should get off the highway and go horizontally (u = 1) to the end point(z_1, w).

If the initial(z_0, w), and the end point(z_1, w)lie under the parabola, but$w < w \le z_0^2 < z_1^2$you don't have to go to the highway. The optimal trajectory consists of vertical and horizontal segments. From starting point(z_0, w)should move vertically to a point(z_0, w), and then from it horizontally - to the end point(z_1, w).

Since knowledge and skills can only be accumulated during training, the end point is always located not lower and not to the left of the starting point. If they lie on the same horizontal or vertical line, then the optimal trajectory consists of one horizontal or, respectively, vertical segment.

We have analyzed one case of the location of the start and end points relative to the parabola. There are three more:

(1) start point below the parabola, end point above;

(2) both points are above the parabola;

(3) the start point is above the parabola, the end point is below.

These cases can be analyzed similarly to the one above for the situation where both points are below the parabola. First you need to go to the highway - if the starting point is below the parabola, then vertically, if higher - horizontally. Then move along the highway and get off it so as to get to the end point horizontally (if the end point is below the parabola) or vertically (if above).

The described procedure for constructing the optimal trajectory resembles the natural behavior of a motorist - first, get to the highway as quickly as possible, drive the main part of the way along it, and then turn off the highway at the right time and get to the end point in the shortest way. Therefore, the "special solution" along which one must move most of the time is called the highway.

Let us turn to Problem 2. In it, from the family of optimal trajectories leading from the initial point (z0; w0) to different points of the ray (z1; w1), w0 < w1 < +∞, we must choose the trajectory that requires the minimum time. It turns out that for z1 < 2z0 on the ray the point with w1 = z0 (z1 – z0) is optimal, the trajectory consists of vertical and horizontal segments. If you want to significantly increase the amount of knowledge, ie. for z1 > 2z0, then the optimal is$w_1 = z_1^2/4$, while the main part of the optimal trajectory passes along the highway w = z2 from the point$(z_0; z_0^2)$to the point$(z_1/2\,; z_1^2/4)$. Consequently, the more knowledge z1 needs to be mastered, the greater the proportion of time you need to move along the highway, while giving 2/3 of the time to increasing skills and 1/3 of the time to accumulating knowledge, and at the end of the learning period, only increase knowledge without wasting study time for skill development.

The discussion of the results

The value u = 1/3 obtained for the main section of the optimal learning trajectory can be interpreted as follows: when driving along the highway, i.e. during the main period of study, there should be two seminars per lecture, 45 minutes of explanation (academic hour) - 90 minutes of problem solving (two academic hours).

In the initial period of training, in order to enter the highway, an optimal balance of knowledge and skills should be achieved. If there is enough knowledge, but few skills (for example, due to the lack of regular training in solving problems), the teacher needs to organize classes to develop the necessary skills. If, on the contrary, the skills are developed to the required extent, but there is little knowledge, then the teacher should achieve an increase in the volume of the student's knowledge. Thus, at the beginning of teaching the discipline, the teacher must ensure that all students reach the ratio of knowledge and skills required for mastering the discipline. From the foregoing, it follows that it is expedient to conduct a lesson at the beginning of the course devoted to the repetition of the main concepts used in the future and the awakening of the relevant skills.

Actions upon completion of training are determined by the task. In accordance with the above, you should get off the highway. In the practical solution of problem 1, it is necessary to reach the target indicators. Depending on the values of the coordinates of the end point, we are talking about either the necessary development of skills, or the acquisition of knowledge that complements the knowledge obtained during the study of the main part of the course.

The above solution of problem 2 leads to the conclusion that the distribution of study time between lectures and seminars should change dramatically at the final stage of education. All the time should be given to lectures. At the final stage, students acquire the same amount of knowledge as in all previous studies. But at the same time, they do not waste time on developing skills, since they received the necessary skills by the beginning of the final stage of training.

When driving on the highway, i.e. during the main period of the educational process, the optimal distribution of time between explanations and problem solving is the same for all students, regardless of the individual coefficients k1 and k2 (see the system of equations (1) - (2)). This fact of stability of the optimal solution (in the sense disclosed in [8]) shows the possibility of organizing learning that is optimal for all students at the same time. Indeed, the specific values of the coordinates of the start and end points do not affect the optimal distribution of time during the main training period. In this case, the time of movement to the exit to the highway depends, of course, on the initial position (x0; y0) and individual coefficients k1 and k2.

The results obtained in the mathematical model are consistent with empirical ideas about the optimal organization of the educational process and the practical experience of the author as a teacher. Naturally, at the beginning of the training period, it is necessary to adjust the level of knowledge and skills of students, so to speak, "bring them to a common denominator", with the aim of their learning as part of a single stream (and not individually). Then comes the "movement along the highway": lectures are read for the entire stream, seminars are held for groups of students.

Let's focus on the final period of training. In our opinion, it differs significantly from the main period. Let's discuss the organization of training for undergraduates. Is it necessary, like students of previous courses, to aim them at solving specific problems by one method or another, as is usually done at seminars? The fact is that there are a lot of types of tasks, as well as methods, and in any case, students will master only a small part of the intellectual tools developed to date. It is more useful to give undergraduates bird's eye views of a number of topics, leaving the details for independent study by those graduates who will need them in their practical work. This is how he built teaching undergraduates the discipline "Organizational and economic modeling" (see [9-11]).

In addition to the general strategy for organizing the educational process, the model determines the numerical values of the share of time spent on improving knowledge (it turned out to be equal to 1/3), and the share of material (1/2) presented in the final lectures without working out at seminars (it turned out to be equal to 1/ 2). These numerical values are quite consistent with the practical experience of the author in teaching various disciplines.

The scientific results of this article are applicable not only when discussing the problems of teaching in higher education, but also for organizing the educational process of students in secondary schools. It is for the second case that the initial approaches to the construction of mathematical models for the optimal control of the learning process [12, 13], developed in this article, were outlined. The initial ideas received some further development in [1, 14].

Conclusion

The sketch model of the learning management process (1) - (2) and its modification (3) made it possible to obtain a number of practically useful recommendations, including those expressed in numerical form. At the same time, it was not necessary to clarify the methods for measuring the amount of knowledge and skills available to the student. It was enough to agree that these quantities satisfy the qualitative relations leading to equations (1) and (2).

In accordance with the proposed innovative mathematical model of optimal control of the learning process, it is recommended to first enter the highway, i.e. to achieve the optimal ratio of the initial levels of knowledge and skills of each student. During the main period of the educational process, one should move along the highway, i.e. one third of the time should be devoted to lectures, two thirds to seminars. It is important that this recommendation is optimal for all students at the same time. The final stage is different for the two tasks. If it is necessary to achieve predetermined levels of knowledge and skills (task 1), then at a certain point in time one should leave the highway and complete the learning process either by increasing knowledge or developing skills, depending on what was achieved during the main period of study.

The possibilities of practical application of the recommendations obtained on the basis of the innovative mathematical model (1) - (2) deserve further discussion.

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