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01.00.00 Математические и компьютерные науки

О МАТЕМАТИЧЕСКОЙ МОДЕЛИ ДИНАМИКИ РЕЗУЛЬТАТИВНОСТИ ИСПОЛЬЗОВАНИЯ ПРЕЗЕРВАТИВОВ И ТЕРАПЕВТИЧЕСКОГО ЛЕЧЕНИЯ ВИЧ/СПИДА

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Так как до сих пор отсутствуют эффективные средства лечения вирусного иммунодефицита человека (ВИЧ) и вирусного синдрома приобретенного иммунодефицита (СПИД) с момента их появления, то многие научные исследования направлены на построение математических моделей, которые моделируют возможности предупреждения, профилактики и ликвидации этой болезни. В данной работе формулируется математическая модель, которая исследует динамику влияния одновременного применения презерватива и терапевтического лечения, как средства (инструмент) против распространения ВИЧ/СПИДа в гетеросексуальной популяции. В предлагаемой модели используется нелинейные дифференциальные системы, состоящие из семи (7) дифференциальных уравнений в семи (7) группах населения. В модели учитывается уровень рождаемости изучаемого населения, а также доля инфицированных мужчин, которые одновременно использует презерватив и антиретровирусную терапию. Модель исследует поведенческие изменение пропорций инфицированных индивидов населения после применения мер регулирования (использование презервативов и антиретровирусной терапии). В работе доказано, что эффективность профилактических мер в значительной мере зависит от ряда описанных параметров. Кроме того, результаты численных экспериментов показали, что при отсутствии профилактических мер инфекция охватывает всё население. Исследование влияния исходных параметров модели показывает, что население с высоким уровнем использования презервативов, при наличии антиретровирусной терапии, как средство профилактики, значительно снижает уровень ВИЧ/СПИДа. Таким образом, степень

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01.00.00 Mathematical and Computer Sciences

ON MATHEMATICAL MODEL OF THE DYNAMICS OF THE IMPACT OF CONDOM USE AND THERAPEUTIC TREATMENT OF HIV/AIDS

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Following the absence of a definite treatment for the human immunodeficiency virus (HIV) or the acquired immune deficiency syndromes (AIDS) since their appearance, many scientific studies with the help of mathematical models have been formulated to the extent possible to prevent and eradicate the disease. In this article we have formulated a mathematical model that explores the dynamics of the impact of the use of condom and therapeutic treatment simultaneously, as a means (tools) against the spread of HIV/AIDS in the heterosexual population. The proposed model uses a nonlinear differential equation system consisting of seven (7) differential equations in seven (7) groups of the population. The model takes into account natural birth rate of the studied population, and the proportion of infected males, which simultaneously uses condom and antiretroviral therapy. The model explores the behavioral change of proportion of infected individuals in the population following the application of control measures (condom use and antiretroviral therapy). It is proved that the effectiveness of preventive measures greatly depends on a number of parameters described. In addition, the results of numerical experiments showed that in the absence of both preventive measures, the entire population is contaminated with the infection. The interaction of the model parameters show that the population with high levels of condom use in the presence of significant adherence to antiretroviral therapy as prophylaxis significantly reduces the level of HIV/AIDS. Thus, prevention of infection is significantly improved with the increasing number of the infected population using condoms and antiretroviral therapy simultaneously распространения инфекции значительно снижается с увеличением доли числа индивидов инфицированного населения, которые используют презервативы и антиретровирусные терапии одновременно

Ключевые слова: ИСКОРЕНЕНИЕ, ВОЗДЕРЖАНИЕ, ПРЕДОТВРАТИМЫХ, АДЕПТ, ИММУННЫЙ, ТЕРАПЕВТИЧЕСКИ, ГЕТЕРОГЕННЫЙ, МОДЕЛИРОВАНИЕ, ЗАРАЖЕННЫЕ, ЭПИДЕМИЯ, ПРОФИЛАКТИКА

Keywords: ERADICATION, ABSTINENCE, PREVENTABILITY, ADHERENT, IMMUNE, THERAPEUTIC, HETEROGENEOUS, SIMULATION, CONTAMINATED, EPIDEMICITY, PROPHYLACTICS

1. INTRODUCTION

The ravaging of the society by several numbers of infectious diseases has for long thrown Scientist and Researchers into an atmosphere of constant search for possible solutions. The Human Immune Deficiency Virus (HIV), which is the basic route of the deadly disease called Acquired Immune Deficiency Syndrome (AIDS), have been branded the greatest and the most complicated health menace of all and including this 21st century.

In the context of its complex and perilous nature, several research works have been carried out as far back as 1987, when [1], developed simple function for the growth in the number of individuals who will develop AIDS and for the distribution of incubation period of those individuals [2], yet, at present, no known cure for HIV/AIDS. Rather, to be able to contained the widespread of this deadly disease, Scientist and Researchers have resorted to the use and application of alternative control measures ranging from awareness campaigns on the practice of sexual intercourse with uninfected mutual partner, abstinence from unprotected sex, counseling, Antiretroviral Therapy (ART) and having protected sex with condom [3][4].

Tackling the scourge posed by HIV required multi-dimensional approach which can be achieved via formulation of mathematical models. Mathematical modeling is known for its power to substantially inform and enhance social and behavioral HIV prevention research [5]. Modeling of HIV transmission is geared towards extracting as much information as possible from available data in order to provide an accurate representation of both the knowledge and uncertainty about the epidemic [6]. Notable works in this direction include those by [7, 8, 9, 10, 11, 12].

The motivating factor for this present study is that by [4]. Formulated in that study, were the transmission dynamics of HIV/AIDS in a two-sex population considering Counseling and Antiretroviral Therapy (ART). The impact of growing access to ART on the consistent use of condom is of paramount importance and remains an ongoing debate among researchers [13, 14, 15]. Anti-retro-viral therapy (ART) is the placement of infected individuals on antiretroviral drugs in correct manner, with adherence support [5]. It is a biological treatment that depresses the Immune system, thereby prolonging the life-span of the case study.

Furthermore, unlike the earlier model by [3], which studied the impact of non-compliance by the population in condom use for the prevention of the spread of HIV/AIDS, as a single strategy approach, this present paper formulate a mathematical model which accounts for the dynamics of the impact of simultaneous condom use and therapeutic treatment (ART) of HIV/AIDS in a heterogeneous population. The present model encompasses novelty features which includes the natural birth rate defined on mutual interactions between the males and females population as well as the proportion of infected males that simultaneously uses both the condom and ART. This later factor also account for the variation of this present model and that by [5].

Several independent studies on the dynamics of condom use as well as antiretroviral therapy (ART) in the prevention of HIV/AIDS infection have been conducted. But how effective is the simultaneous use of both the condom and ART (combined treatment) as a means of preventability, have not been adequately exhausted. Therefore, this present study, which accounts for the dynamics of the impact of simultaneous condom use and therapeutic treatment (ART) in the prevention and eradication of HIV/AIDS, came at the appropriate period as the search for accelerated approach to the eradication of HIV/AIDS infection is given much more attention. It is hope that the present work will provide insight into the potential impacts of condom use with antiretroviral therapy in the control and eradication of the spread of HIV/AIDS.

2. PARAMETERS AND MODEL EQUATIONS

Formulation of the mathematical model for this study, took into account the following assumptions:

i. Population under study is heterogeneous;
ii. Infection is either horizontal or vertical;
iii. Infected individual die natural death or due to infection;
iv. Efficacies of both control measures (the condom and ART) varies;
v. Only infected males use the condom;
vi. Both infected males and females use the control measure – ART;
vii. Age-structure is ignored;
viii. Treatment is not life-span dependent.

Other assumptions by [4], are equally observed. These assumptions are essential for the structuring of the model flow-chart as seen in figure 1 below:

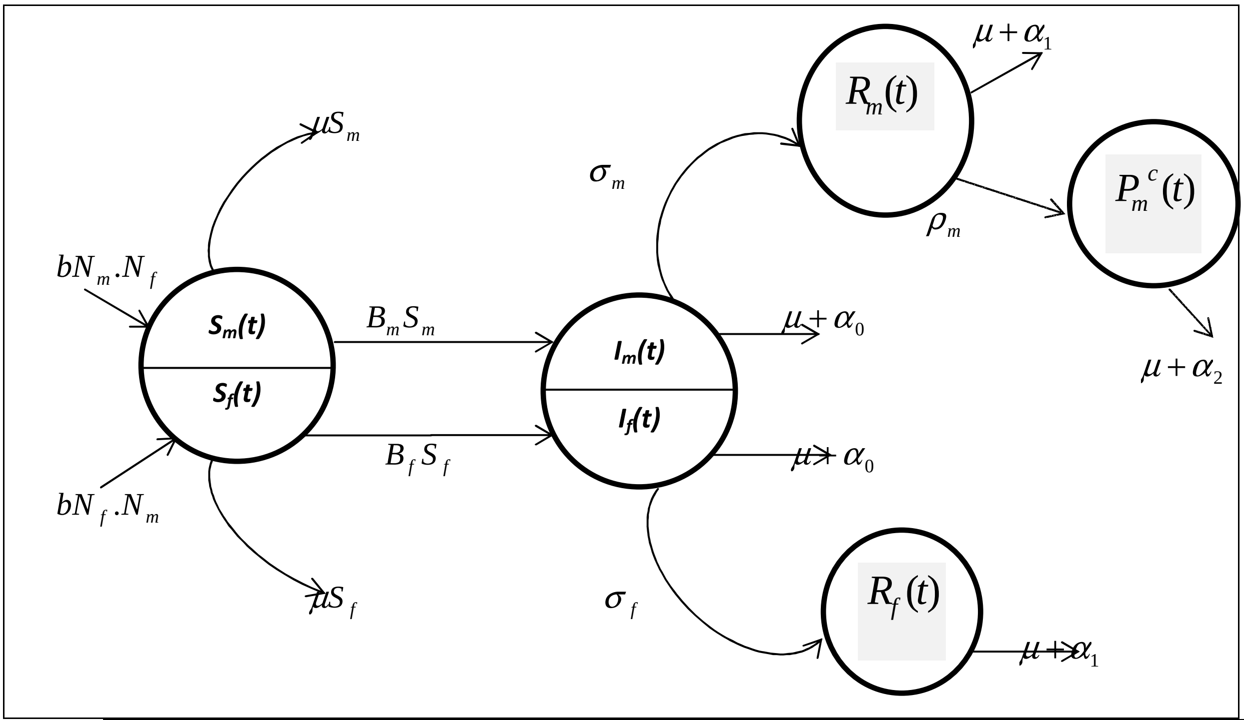

Figure 1: A differential flow-Chart of the model of the impact of condom use and antiretroviral therapy in prevention of HIV/AIDS

From fig.1 above, the parameters (functions) of the model are defined as follows:

$S_m(t)$ - Number of susceptible males at time t;

$I_m(t)$ - Number of infected males at time t;

$R_m(t)$ - Number of infected males who use ART at time t;

$P_m^{\,c}(t)$ - Number of infected males that use both ART and condom;

$c_m(t)$ - Average number of sexual contacts by infected males with females per unit time;

$c_m^{\,1}(t)$ - Average number of sexual contacts by infected males who use ART with females per unit time;

$c_m^{\,2}(t)$ - Average number of sexual contact of infected males that use both ART and condom with females per unit time t;

β_m - Probability of transmission by an infected male;

$\beta_m^{\,1}(t)$ - Probability of transmission by an infected male who use ART;

$\beta_m^{\,2}(t)$ - Probability of transmission by an infected male that use both ART and the condom;

σ_m - Proportion of infected males who use ART per unit time;

ρ_m - Proportion of infected males who use both ART and the condom per unit time;

$B_m(t)$ - The infection rate of males at time t (incidence rate of males);

where,

$$N_m = S_m + I_m + R_m + p^c{}_m$$

$S_f(t)$ - Number of susceptible females at time t;

$I_f(t)$ - Number of infected females at time t;

$R_f t)$ - Number of infected females who use ART at time t;

$c_f(t)$ - Average number of sexual contacts by infected females with males per unit time;

$c_f^{\ 1}(t)$ - Average number of sexual contacts by infected females receiving
ART with males per unit time;

β_f - Probability of transmission by an infected female;

$\beta_f^{\ 1}$ - Probability of transmission by an infected female who use ART;

σ_f - Proportion of infected females who use ART per unit time;

$B_f(t)$ - The infection rate of females at time t (incidence rate of females);

where,

$$N_f = S_f + I_f + R_f$$

$b.N_m.N_f$ - Population birth rate, $b \geq 0$

μ - Natural death rate, $\mu \geq 0$

α_0 - Population death rate of infected without treatment, $\alpha_0 \geq 0$

α_1 - Population death rate of infected who use only ART, $\alpha_1 \geq 0$

α_2 - Population death rate of infected who use both ART and the condom, $\alpha_2 \geq 0$.

From the flow-chart (fig. 1) and the parameters stated above, the coefficients are considered positive and the signs before the terms are taken with plus if the arrow enters the area and with minus sign, if the arrow goes out of the scope [3]. Essentially, the model also took into account, natural birth rate $b.N_m.N_f$, natural death rate μ; and population death rate of those without treatment (α_0), receiving only ART (α_1) and those using both ART and condom

(α_2) respectively. Thus, the model is governed by the following derived non-linear ordinary differential equations:

$$\frac{dS_m}{dt} = bN_m.N_f - B_m S_m - \mu S_m \tag{2.1}$$

$$\frac{dI_m}{dt} = B_m S_m - (\mu + \alpha_0 + \sigma_m) I_m \tag{2.2}$$

$$\frac{dR_m}{dt} = \sigma_m I_m - (\mu + \alpha_1 + \rho_m) R_m \tag{2.3}$$

$$\frac{dP^c{}_m}{dt} = \rho_m R_m - (\mu + \alpha_2) P^c{}_m \tag{2.4}$$

In a similar approach, we derive the model equations for the female population as:

$$\frac{dS_f}{dt} = bN_f.N_m - B_f S_f - \mu S_f \tag{2.5}$$

$$\frac{dI_f}{dt} = B_f S_f - (\mu + \alpha_0 + \sigma_f) I_f \tag{2.6}$$

$$\frac{dR_f}{dt} = \sigma_f I_f - (\mu + \alpha_1) R_f \tag{2.7}$$

Differential equations for the total population of males and females is supplemented by algebraic equations

$$N_m = S_m + I_m + R_m + p^c{}_m \tag{2.8}$$

$$N_f = S_f + I_f + R_f \tag{2.9}$$

From the investigation by [16], the incidence rates of infection denoted by B_m and B_f are expressed by the relative $\frac{I_f}{N_f}, \frac{R_f}{N_f}$ and $\frac{I_m}{N_m}$, $\frac{R_m}{N_m}, \frac{P^c m}{N_m}$ of HIV-infected individuals:

$$B_m = c_m \beta_f \frac{I_f}{N_f} + (c_m{}^1 + c_m{}^2) \beta_f{}^1 \frac{R_f}{N_f} \tag{2.10}$$

$$B_f = c_f \beta_m \frac{I_m}{N_m} + c_f{}^1 \left(\frac{\beta_m{}^1 R_m + \beta_m{}^2 P^c{}_m}{N_m} \right) \tag{2.11}$$

From equations (2.10) and (2.11), if we let $c_m{}^2$ and $\beta_m{}^2$ equal zero, then we obtain the model by [4]. Therefore, the present model is enhance by the parameters $P_m{}^c(t), c_m{}^2(t), \beta_m{}^2(t), \rho_m, \alpha_2$ and equations (2.10) – (2.11) as against that by [4].The original data are summarized in table 1 below:

Table 1: Summarized original parameter data:

Symbols	**Expression for the males**	**Symbols**	**Expression for the females**
S_m	number of Susceptible males at time *t*;	S_f	number of susceptible females at time *t*;
I_m	number of infected males at time *t*;	I_f	number of infected females at time t;
R_m	number of infected males who use the ART at time t;	R_f	number of infected females who use the ART at time t;
$P_m{}^c$	number of infected males that use both ART and condom;	$c_{f(t)}$	average number of sexual contacts by infected females with males per unit time;
$c_{m(t)}$	average number of sexual contacts by infected males with females per unit time;	$c_f{}^1{}_{(t)}$	average number of sexual contacts by infected females who use ART with males per unit time;
$c_m{}^1{}_{(t)}$	average number of sexual contacts by infected males who use ART with females per unit time;	N_f	$S_f(t) + I_f(t) + R_f(t)$ - total population of females at time t;
$c_m{}^2{}_{(t)}$	average number of sexual contacts by infected males who use both ART and condom with females per unit time;	β_f	Probability of transmission by an infected female;
N_m	$S_m(t) + I_m(t) + R_m(t) + P^c{}_m(t)$ - total population of males at time t;	$\beta_f{}^1$	Probability of transmission by an infected female who use ART;
β_m	Probability of transmission by an infected male;	σ_f	the proportion of infected females who use ART per unit time;
$\beta_m{}^1$	Probability of transmission by an infected male who use ART;	$B_f(t)$	the infection rate of females at time t, (incident rate);
$\beta_m{}^2$	Probability of transmission by an infected male who use both ART and the condom;	$b.N_f.N_m$	natural birth rate of females population, $b \geq 0$;
σ_m	the proportion of infected males who use ART per unit time;	μ	natural death rate, $\mu \geq 0$;
ρ_m	the proportion of infected males who use both ART and the	α_0	Population death rate of infected Who receive ART, $\alpha_0 \geq 0$;

	condom per unit time;		
$B_{m(t)}$	the infection rate of males at time t, (incident rate);	α_1	Population death rate of infected without ART $\alpha_1 \geq 0$;
$b.N_m.N_f$	natural birth rate of male population, $b \geq 0$;		
μ	natural death rate, $\mu \geq 0$;		
α_0	Population death rate of infected Who receive ART, $\alpha_0 \geq 0$;		
α_1	Population death rate of infected without ART $\alpha_1 \geq 0$;		
α_2	Population death rate of infected Who use both ART and the condom, $\alpha_2 \geq 0$;		

3. TRANSFORMATION OF MODEL EQUATIONS AND ANALYSIS

In this section, we shall be involve in the transformation of the model equations into dimensionless form, then followed by derivation of population changes caused by infection at a given period of time. It is worthy to note that the consequence of equation transformation into proportions is based on the following:

i. It reduces the numbers of seeming complex equations for easy handling;

ii. It initiate the biological meanings of the proportions of infected individuals;

iii. The prevalence of infection is defined.

Let $N_m(0) = N_m, \; N_f(0) = N_f$

Then

$$s_m = S_m / N_m \tag{3.1}$$

$$y_m = I_m / N_m \tag{3.2}$$

$$r_m = R_m / N_m \tag{3.3}$$

$$p_m = P_m{}^c / N_m \tag{3.4}$$

$$S_f = S_f / N_f \tag{3.5}$$

$$y_f = I_f / N_f \tag{3.6}$$

$$r_f = R_f / N_f \tag{3.7}$$

In terms of proportions,

$$m(t) = N_m(t) / N_m = s_m + y_m + r_m + p_m \tag{3.8}$$

and

$$f(t) = N_f(t) / N_f = s_f + y_f + r_f \tag{3.9}$$

Furthermore, the coefficients (2.10) – (2.11), in its dimensionless form are transformed into:

$$B_m = c_m \beta_f \frac{y_f}{f(t)} + (c_m{}^1 + c_m{}^2) \beta_f{}^1 \frac{r_f}{f(t)} \tag{3.10}$$

$$B_f = c_f \beta_m \frac{y_m}{m(t)} + c_f{}^1 \left(\frac{\beta_m{}^1 r_m + \beta_m{}^2 p_m}{m(t)} \right) \tag{3.11}$$

Then, from equations (2.1) – (2.4), the transformations in the infection process are derived as:

$$s_m' = bm(t).f(t) - B_m s_m - \mu s_m \tag{3.12}$$

$$y_m' = B_m s_m - (\mu + \alpha_0 + \sigma_m) y_m; \tag{3.13}$$

$$r_m' = \sigma_m y_m - (\mu + \alpha_1 + \rho_m) r_m; \tag{3.14}$$

$$p_m' = \rho_m r_m - (\mu + \alpha_2) p_m \tag{3.15}$$

In a similar notion, equations (2.5) – (2.7) can be obtained as:

$$s_f' = bf(t).m(t) - B_f s_f - \mu s_f \tag{3.16}$$

$$y_f' = B_f s_f - (\mu + \alpha_0 + \sigma_f) y_f \tag{3.17}$$

$$r_f' = \sigma_f y_f - (\mu + \alpha_1) r_f \tag{3.18}$$

In a simplified form, the various changes in each of the population groups are simulated and analyzed in terms of equations (3.12) – (3.18) as summarized in Table 2, below:

Table 2: Proportions of the dynamics of HIV control measures

Group	Derivatives	*Eqn. no.*
S_m	$s_m' = bm(t).f(t) - B_m s_m - \mu s_m$	(3.12)
I_m	$y_m' = B_m s_m - (\mu + \alpha_0 + \sigma_m) y_m$	(3.13)
R_m	$r_m' = \sigma_m y_m - (\mu + \alpha_1 + \rho_m) r_m$	(3.14)
$p^c{}_m$	$p_m' = \rho_m r_m - (\mu + \alpha_2) p_m$	(3.15)
S_f	$s_f' = bf(t).m(t) - B_f s_f - \mu s_f$	(3.16)
I_f	$y_f' = B_f s_f - (\mu + \alpha_0 + \sigma_f) y_f$	(3.17)
R_f	$r_f' = \sigma_f y_f - (\mu + \alpha_1) r_f$	(3.18)

4. NUMERICAL SIMULATION AND ANALYSIS

The model represents a set of 7 non-linear ordinary differential equations involving 7 different groups of the population ($s_m, y_m, r_m, p_m, s_f, y_f, r_f$) and are simulated using hypothetical values as generated in Table 3, below:

Table 3: Table of parameter values for variants 1 - 4

Variant	b	μ	α_0	α_1	α_2	σ_m	σ_f	ρ_m	c_m	$c_m^{\ 1}$	$c_m^{\ 2}$	c_f	$c_f^{\ 1}$	β_m	$\beta_m^{\ 1}$	$\beta_m^{\ 2}$	β_f	$\beta_f^{\ 1}$
1.	0.02	0.1	0.2	0.15	0.15	0	0	0	5	5	5	5	2	0.2	0.1	0.05	0.2	0.1
2.	0.02	0.1	0.2	0.15	0.15	0.2	0	0	5	5	5	5	2	0.2	0.1	0.05	0.2	0.1
3.	0.02	0.1	0.2	0.15	0.15	0.2	0.2	0	5	5	5	5	2	0.2	0.1	0.05	0.2	0.1
4.	0.02	0.1	0.2	0.15	0.15	0.2	0.2	0.2	5	5	5	5	2	0.2	0.1	0.05	0.2	0.1

where the initial values of all characteristics are as in table 4.

Table 4: Initial values of proportions of the population

Proportions	$s_m(0)$	$y_m(0)$	$r_m(0)$	$p_m(0)$	$s_f(0)$	$y_f(0)$	$r_f(0)$
Values	0.7	0.1	0.1	0.1	0.6	0.2	0.2

For simulation compatibility, we rewrite the parameters ($s_m, y_m, r_m, p_m, s_f, y_f, r_f$) as a vector quantity, i.e.

$$A = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7\}^T = \{A_i\}_{i=1}^7$$

$$\{A_m\} = \sum_{i=1}^{4} A_i\,,\ \{A_f\} = \sum_{i=5}^{7} A_i\,.$$

Therefore, the systems of differential equations can be rewritten as:

$$\begin{cases} \dfrac{dA}{dt} = f(A),\ t \in [0,T] \\ A(0) = A_0 \end{cases}.$$

The differentiation of the above system of equations is done using one of the several numerical methods. Here, we use the MATHCAD [3], which is known for its in-built function “rkfixed” realizing Runge-Kutter method of accuracy of order 4.

In the analysis that follows, we bear in mind that our investigation is the behavioral change of the proportions of the population of infected groups (r_m, p_m, r_f) following the application of the control measures (condom use and ART). The efficacies of the control measures (condom use and ART) are predominantly determined by $\sigma_m, \sigma_f, \rho_m, \beta_m^{\ 1}, \beta_m^{\ 2}, \beta_f^{\ 1}$. This does not play down on the importance of other parameters such as $c_m^{\ 1}, c_m^{\ 2}, c_f^{\ 1}$.

Now, if $J^{\langle 1 \rangle}$ represent the time interval t, and from Table 2, $J^{\langle 2 \rangle}$,…., $J^{\langle 8 \rangle}$ represent $s_m, y_m, r_m, p_m, s_f, y_f, r_f$ respectively, then the corresponding graphs taking from Tables(3 & 4), are as in figures (2 – 5) below. Moreover, in each of figures (2 – 5), the investigations were carried out by the highlighted parameters as in variants (1 – 4) of table 3, which predominantly affects the treatment factors.

Note: The analysis that follows shall be express in terms of years of survival by various proportions of the population understudy.

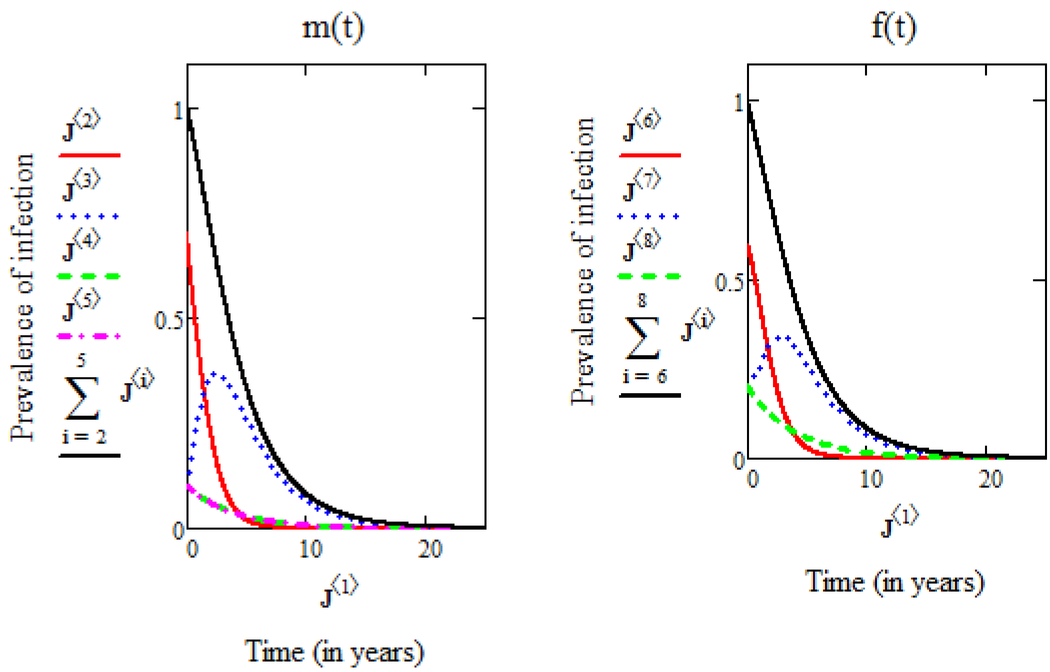

Fig.2: Graphical simulations of $s_m, y_m, r_m, p_m, s_f, y_f, r_f$ from model (3.12) – (3.18) against time with $J^{\langle i \rangle}$ - the sum of male and female proportions. Parameter values are in variant (1) - Table 3.

As a control measure over our studies, from variant 1 (table 3), we simulate as in figure 2, above, the situation where nobody use any of the preventive measures (condom use and ART), with 10% natural death rate (i.e. $\sigma_m, \sigma_f, \rho_m = 0, \mu = 0.1$). Putting contact rate of 20% by infected females who ought to be on ART treatment and low transmission rate of 5% by infected males who ought to be on both condom use and ART, the proportions of s_m and s_f were early contaminated with infection in less than 10 years. Infection increases for both male and female population (y_m, y_f), leading to entire extinction of the population within 20 years. Numerically, the behavioral changes can be expressed as follows: $s_m \le 8, y_m \le 20, r_m \le 15, p_m \le 10$, and $s_f \le 8, y_f \le 20, r_f \le 15$. This shows that the limit to which the entire population survives can be expressed by

$$\{A_m\} = \sum_{i=2}^{5} A_i \le 20 \text{ and } \{A_f\} = \sum_{i=6}^{8} A_i \le 20 .$$

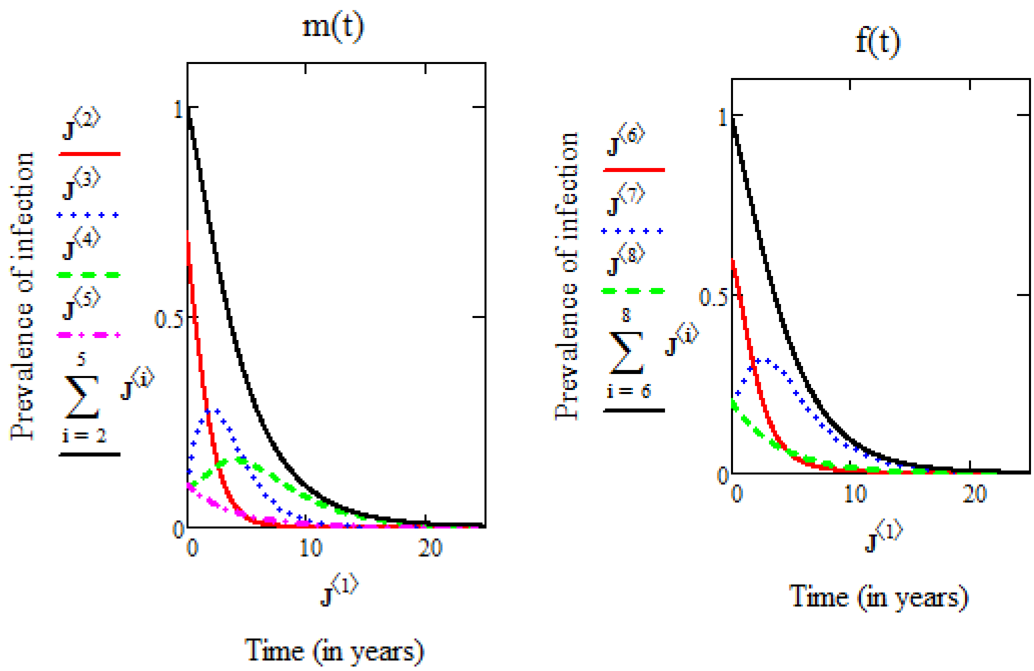

Fig.3: Graphical simulations of $s_m, y_m, r_m, p_m, s_f, y_f, r_f$ from model (3.12) – (3.18) against time with $J^{\langle 1\rangle}$ - the sum of male and female proportions. Parameter values are in variant (2) - Table 3.

From figure 3 above, setting the parameter values of fig. 2 as bases, we conduct an investigation with only 20% of infected males using only ART treatment factor (i.e. $\sigma_m = 0.2, \sigma_f = 0, \rho_m = 0$). Application of ART as a preventive measure by infected males leads to retardation of infection and prolong lifespan of about 25 years. The impact of this ART is shown by the considerable increase in the number of years before contamination of the susceptible by infection compared to that of fig. 2. The investigation indicates $s_m \le 8, y_m \le 12, r_m \le 20, p_m \le 10$ and $s_f \le 15, y_f \le 20, r_f \le 12$, with visible extinction period of the entire population given by

$$\{A_m\} = \sum_{i=2}^{5} A_i \le 20 \text{ and } \{A_f\} = \sum_{i=6}^{8} A_i \le 20.$$

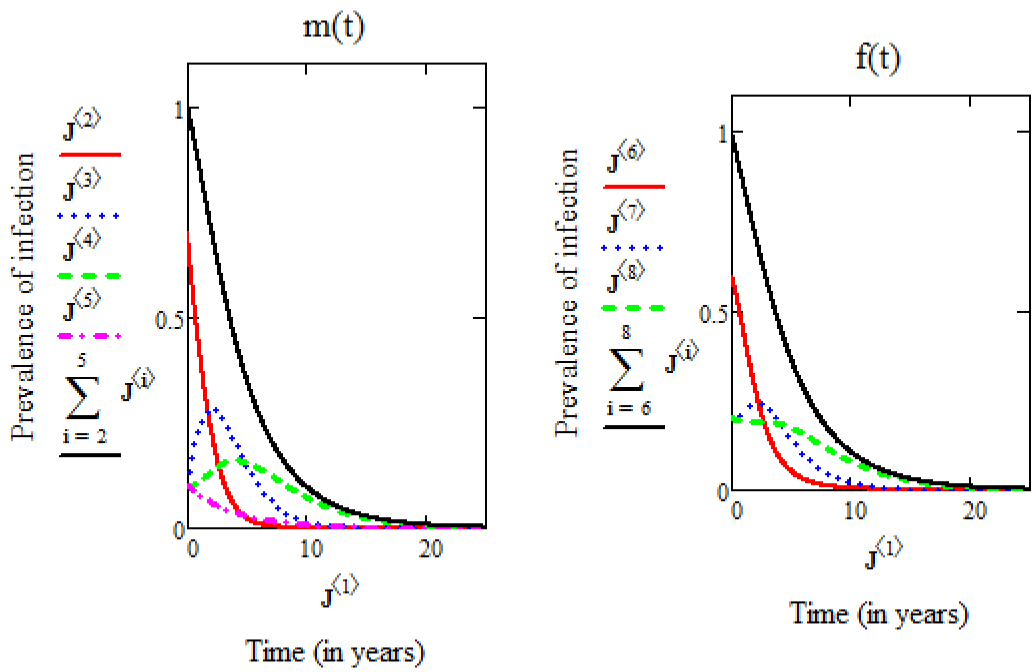

Fig.4: Graphical simulations of $s_m, y_m, r_m, p_m, s_f, y_f, r_f$ from model (3.12) – (3.18) against time with $J^{\langle 1\rangle}$ - the sum of male and female proportions. Parameter values are in variant (3) - Table 3.

Figure 4 above, represent a slight modification of variant 2, resulting to variant 3 (table 3). Here, we studied only the impact of the use of ART by both infected male and female population, as a single treatment factor (i.e. $\sigma_m = \sigma_f = 0.2, \rho_m = 0$). Again, contact rates of population proportions observing treatment were on the average with low transmission rates (i.e. $c_m^{\ 1} = 5, c_f^{\ 1} = 2; \beta_m^{\ 1}, \beta_f^{\ 1} = 0.1, \beta_m^{\ 2} = 0.05$). We see that the limits of survival for the various proportions are defined by the intervals $s_m \le 8, y_m \le 12, r_m \le 20, p_m \le 10$ and $s_f \le 10, y_f \le 12, r_f \le 20$. The extent to which the entire population survives lies in the range,

$$\{A_m\} = \sum_{i=2}^{5} A_i \le 20 \text{ and } \{A_f\} = \sum_{i=6}^{8} A_i \le 20 .$$

Finally, from fig. 5 below, we intensify our investigation as compared to figure 4. Here, various proportions of the population understudy were exposed to ART only and both condom use with ART as preventive measures (i.e. $\sigma_m, \sigma_f, \rho_m = 0.2$) with infected males maintaining average contact rate and females, low sexual contact rate ($c_m{}^1 = c_m{}^2 = 5, c_f{}^1 = 2$). Their respective transmission probabilities were $\beta_m{}^1 = \beta_f{}^1 = 0.1, \beta_m{}^2 = 0.05$. The results shows that $s_m \le 8, y_m \le 12, r_m \le 20, p_m \le 25$ and $s_f \le 10, y_f \le 12, r_f \le 22$. This indicates a survival range of the entire population at

$$\{A_m\} = \sum_{i=2}^{5} A_i \le 25 \text{ and } \{A_f\} = \sum_{i=6}^{8} A_i \le 22.$$

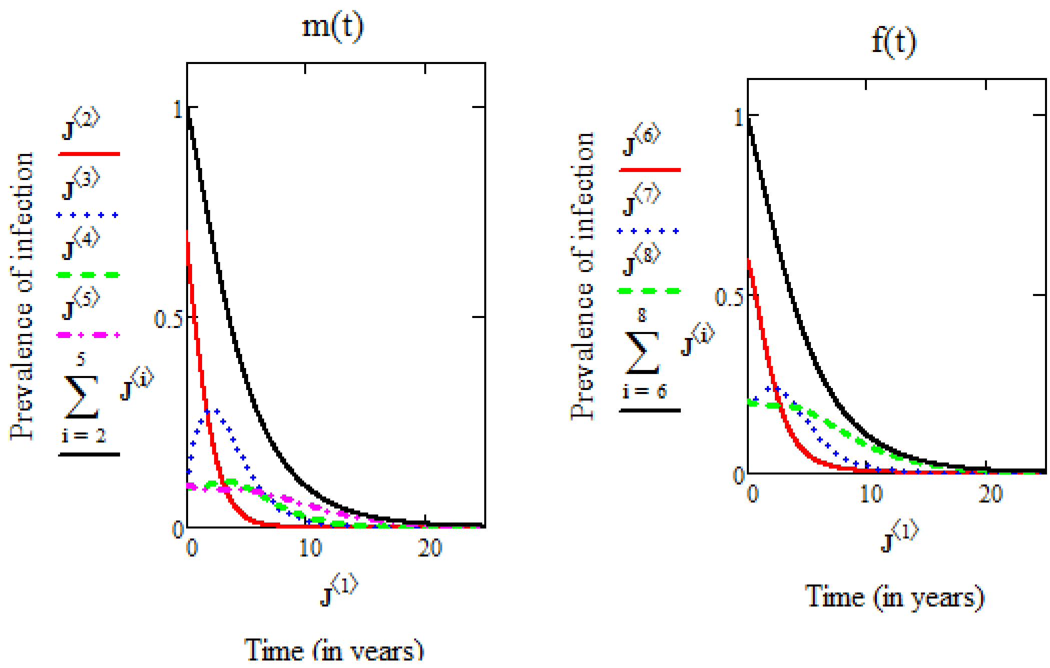

Fig.5: Graphical simulations of $s_m, y_m, r_m, p_m, s_f, y_f, r_f$ from model (3.12) – (3.18) against time with $J^{\langle 1 \rangle}$ - the sum of male and female proportions. Parameter values are in variant (4) - Table 3.

5. DISCUSSION

In this paper, we formulate a mathematical model that studies the dynamics of the impact of ART and simultaneous application of both the condom and therapeutic treatment as preventability of the spread of HIV/AIDS in a heterosexual population. The model uses non-linear differential equation leading to system of seven (7) ordinary differential equations in seven (7) groups of the populations.

From the simulations of our model with the aid of MATHCAD program, the following results were obtained:

i. In the absent of any preventive measure (i.e. no condom and ART),both the susceptible male and female population experienced acute contamination of infection within 8 years, resulting to sharp increase in infection and subsequently, leading to extinction of the entire population within 20 years – fig. 2.

ii. Here only the males use the ART. We see that the proportion of the males that use ART experienced considerable increase in population that live a normal life with life expectancy of over 20 years compared to other proportions of the population. The lifespan of the entire population is been determined by the behavioral changes of this group of the population. ART used by the infected male population exhibit a considerable impact on the female population as they could survive for about 20 years without treatment– fig. 3.

iii. ART reduces the rapid spread of HIV/AIDS infection. But ART without condom use means individuals with higher viral load or low CD4 count before or at the initial of ART have potential to infect their sero-negative sexual partners or at risk of acquiring drug resistant viral strain from their sexual partners who are already infected. Clearly, with the application of only ART by both male and female proportions, we observed reduction and gradual eradication of the spread of HIV/AIDS infection leading to prolonged lifespan and more enhanced normal of about 20 years – fig. 4.

iv. With consistent condom use in the presence of adherent ART (a viral load suppressor), reduction and subsequent eradication of the disease will not only be achieved in a finite time but infected patients live a prolonged life of about 25 years for the males with the two factor treatment and about 22 years for the females with ART as a treatment factor – fig. 5. The use of condom enhances the variation in years as experienced by the male population and as well prolong the lifespan of the female population in the course of interaction.

6. CONCLUSION

Our analysis shows that lack of persistent use of any preventive measures (condom use and ART) as treatment factors increases the spread of HIV – infection. On the other hand, adherent use of ART as a single treatment factor, suppresses HIV/AIDS infection, thereby reducing spread of infection and more importantly, prolonging the lifespan of infected. Furthermore, simultaneous use of the condom and significant adherent to ART, as a means of preventability negatized HIV/AIDS epidemic, suggesting that increase in preventability decreases the level of epidemicity provided the parameters $\sigma_m, \sigma_f, \rho_m$ are sufficiently high. The model therefore, recommends availability and accessibility of preventability measures as a means to the eradication of the spread of HIV/AIDS epidemic. Further studies that would avail the female population the use of condom and ART alongside the male population, is hope to throw more insight on the impact of this bi-therapeutic treatment.

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