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МОДЕЛИРОВАНИЕ МЕТРИКИ АДРОНОВ НА ОСНОВЕ УРАВНЕНИЙ ЯНГА-МИЛЛСА

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В работе рассмотрена система уравнений Янга-Миллса в связи с уравнениями Эйнштейна и Максвелла. Сформулирована модель метрики, удовлетворяющая основным требованиям физики элементарных частиц и космологии

Ключевые слова: АДРОНЫ, МЕТРИКА, ПРОТОН, УРАВНЕНИЯ МАКСВЕЛЛА, УРАВНЕНИЯ ЭЙНШТЕЙНА, УРАВНЕНИЯ ЯНГА-МИЛЛСА

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HADRONS METRICS SIMULATION ON THE YANG-MILLS EQUATIONS

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In this article we consider the Yang-Mills theory in connection with the Einstein and Maxwell equations. The model of a metric satisfying the basic requirements of particle physics and cosmology is proposed

Keywords: EINSTEIN EQUATIONS, HADRONS, MAXWELL EQUATIONS, METRICS, PROTON, YANG-MILLS EQUATIONS.

Introduction

The hypotheses on the global structure of space-time have been formulated in the famous Einstein's paper [1]. Einstein assumed that the universe is stationary, and the average density of matter and the total mass of the universe does not change over time. Friedman [2] has shown that the universe is expanding, it was confirmed by the astronomical data, and also served as a further recognition of general relativity. However, Einstein's theory of gravity was incompatible with Maxwell's equations, so there were numerous attempts to create a unified field theory [3] in the space of five dimensions [4-6]. At present, interest in the classical field theory models waned, and Einstein's equations themselves are primarily used for the solution of cosmological problems.

Due to the development of quantum theory, is a very relevant question of the structure of space-time scale of the proton, as modern lattice models of quantum chromodynamics (QCD) has been successfully used to predict the properties of hadrons [7]. The answer to the question of the structure of space-time can be obtained from the Yang-Mills theory [8-9], which is widely used in particle physics. Connection of the Yang-Mills equations to the Maxwell equations and

Einstein equations as well was established in [10-12]. It has been shown that on 4-manifolds with conformal connection [13], the system of the Yang-Mills equations split into Einstein's equations, Maxwell's equations and the equations of motion of matter. So the necessary prerequisites for a unified field theory have been creating. In this paper we formulated a model of hadrons metric satisfying the basic requirements of particle physics and cosmology.

The basic equations of the model of the cosmological scale

Consider the example of a purely temporary solution of the Yang-Mills equations in the space of torsion-free [11]. We define a metric space as

$$y = h_{ij} w^i w^j = -dt^2 + a^2(t)(dx_1)^2 + b^2(t)(dx_2)^2 + c^2(t)(dx_3)^2 \quad (1)$$

Here $h_{ij} = h^{ij}$ is the metric tensor of the Minkowski space of signature (- + + +),

$$w^1 = dt,\ w^2 = a(t)dx_1,\ w^3 = b(t)dx_2,\ w^4 = c(t)dx_3 \ .$$

Yang-Mills equations can be reduced in this case to Einstein's equations, Maxwell's equations, and equations of motion of matter accordingly, we have

$$b_{11} = \frac{1}{3}\left(\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c}\right) - \frac{1}{6}\left(\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{c}}{bc}\right) \quad (2)$$

$$b_{22} = -\frac{1}{3}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac}\right) + \frac{1}{6}\left(\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}\dot{c}}{bc}\right)$$

$$b_{33} = -\frac{1}{3}\left(\frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{b}}{b} + \frac{\dot{b}\dot{c}}{bc}\right) + \frac{1}{6}\left(\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{c}}{ac}\right)$$

$$b_{44} = -\frac{1}{3}\left(\frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{c}}{bc} + \frac{\ddot{c}}{c}\right) + \frac{1}{6}\left(\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab}\right)$$

$$b_{12}bc = M, \quad b_{34}bc = N$$

$$\dot{b}_{22} + (b_{11} + b_{22})\frac{\dot{a}}{a} = S_2$$

$$\dot{b}_{33} + (b_{11} + b_{33})\frac{\dot{b}}{b} = S_3$$

$$\dot{b}_{44} + (b_{11} + b_{44})\frac{\dot{c}}{c} = S_4$$

$$\dot{S}_2 + S_2\left(\frac{\dot{b}}{b} + \frac{\dot{c}}{c}\right) =$$

$$2b_{12}^2 + 2b_{34}^2 - b_{11}\left(\frac{\ddot{a}}{a} + b_{22} - b_{11}\right) + b_{33}\left(\frac{\dot{a}\dot{b}}{ab} + b_{22} + b_{33}\right) + b_{44}\left(\frac{\dot{a}\dot{c}}{ac} + b_{22} + b_{44}\right)$$

$$\dot{S}_3 + S_3\left(\frac{\dot{a}}{a} + \frac{\dot{c}}{c}\right) =$$

$$-2b_{12}^2 - 2b_{34}^2 - b_{11}\left(\frac{\ddot{b}}{b} + b_{33} - b_{11}\right) + b_{22}\left(\frac{\dot{a}\dot{b}}{ab} + b_{22} + b_{33}\right) + b_{44}\left(\frac{\dot{b}\dot{c}}{bc} + b_{33} + b_{44}\right)$$

$$\dot{S}_4 + S_4\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right) =$$

$$-2b_{12}^2 - 2b_{34}^2 - b_{11}\left(\frac{\ddot{c}}{c} + b_{44} - b_{11}\right) + b_{22}\left(\frac{\dot{a}\dot{c}}{ac} + b_{22} + b_{44}\right) + b_{33}\left(\frac{\dot{b}\dot{c}}{bc} + b_{33} + b_{44}\right)$$

Here $b_{ij} + b_{ji} - 2(h^{ij}b_{ij})h_{ij} = T_{ij}$ is the energy-momentum tensor of matter; M, N - the parameters characterizing the electromagnetic field. Note that the Einstein equations in this notation have the form:

$$b_{ij} + b_{ji} + bh_{ij} = R_{ij} \tag{3}$$

$b = h^{ij}b_{ij}$; R_{ij} is Ricci tensor. Einstein equations (3) can be reduced to the first four equations (2) in the case of the metric (1). In contrast to the standard Einstein equations, they do not contain dimensional parameters characterizing the interaction of the gravitational field with the distribution of matter. This is due to the fact that the quantities in equations (2) - (3) and described by the classical Yang-Mills equations are geometric quantities, like the Ricci tensor in the right-hand side of equation (3).

Some exact solutions and numerical model

In [11] found a particular solution of equations (2) in elementary functions

$$a = \frac{2PQ}{t_0 + t}; \quad b = c = \frac{t + t_0}{2Q}; \quad M^2 + N^2 = \frac{1}{12Q^4};$$
$$b_{11} = \frac{5}{6(t_0 + t)^2}; \quad b_{22} = b_{33} = b_{44} = \frac{1}{6(t_0 + t)^2} \tag{4}$$

Here P, Q, t_0 are the arbitrary constants.

The solution (4) describes a singular time-a process in which the energy density increases at $t \to -t_0$. Among such processes in our universe, we can specify a hypothetical explosion – Big Bang. In this case, the characteristic density of baryonic matter decreases with time, whereas the density of electromagnetic energy remains constant. Note that in the model (2), these densities are diagonal and off-diagonal components of the tensor b_{ij}, respectively.

We point out one particular solution of the system (2)

$$a = a_0 \exp(l\,t), \quad b = B, \quad c = C; \quad M^2 + N^2 = B^2 C^2 \frac{l^4}{12};$$
$$b_{11} = -b_{22} = \frac{l^2}{3}; \quad b_{33} = b_{44} = \frac{l^2}{6} \tag{5}$$

$$y = -dt^2 + a_0^2 e^{2lt}(dx_1)^2 + B^2(dx_2)^2 + C^2(dx_3)^2.$$

Here a_0, B, C, l are the arbitrary constants. The solution (5) gives us an example of a space of constant negative curvature $R = h^{ij} R_{ij} = -2\,l^2$. In this case, the density of baryonic matter and electromagnetic energy density remains constant over time.

Solutions (4-5) were used to adjust the numerical model for the system (2). Explore different modes of transition of the solution (4) to a solution of the type (5) - Fig. 1. We found that if in the initial time we set $b_{11} = const$, then in the next moment, all the diagonal tensor components b_{ij} tend to constant values, and the components of the metric tensor are increasing exponentially - Fig. 1.

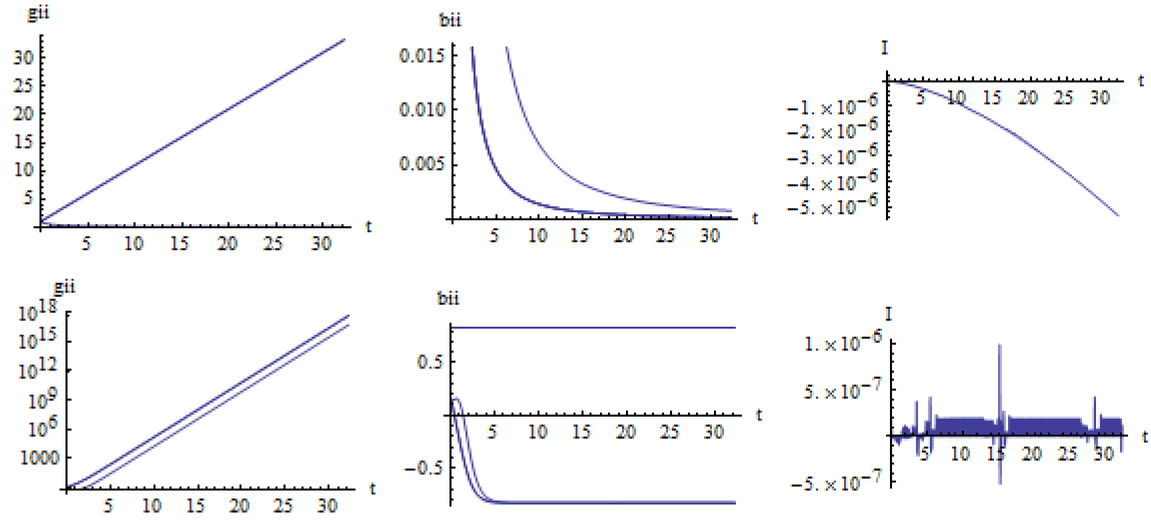

Fig. 1. The exact solution (4) (top) and the transition from the solution (4) to a solution in which all diagonal components b_{ij} tends to a constant (bottom): on the left components of the metric tensor, on the right the accuracy of the solution are shown.

However, only on the solution of the type (5), the density of electromagnetic energy remains constant over time. In all other cases, the density varies according to the equations: $b_{12}bc = M, \quad b_{34}bc = N$.

This disappearance of the electromagnetic field at a large scale is against the astronomical observations, so let's mark solutions (5), as the hypothetical scenario that describes the universe at a large scale.

An alternative scenario is the type (4), but it contradicts the experiments in high-energy physics. Indeed, a decrease in the density of baryonic matter means, among other things, that the proton is unstable. This follows from the fact that the density of the model of baryonic matter and proton density are linked by the fact that the Einstein equation (3) contains only the geometric parameters that describe the density distribution at any scale. However, experimental evaluation of the proton lifetime show that the half-life of a proton is more than 10^{33} years, i.e. greatly exceeds the lifetime of the universe (about 10^{10} years).

For this reason, we should discard the other cosmological models with decreasing density of matter. In this sense, Einstein's hypothesis [1] on the stationary distribution of the density of matter in the universe is correct, however, this hypothesis does not imply that the metric is also stationary. We obtained the solution (5) combines the properties of Einstein's model [1], and Friedman’s model [2] as well, describes the universe as a time-dependent metric, and with a constant density distribution of baryonic matter and electromagnetic field.

Model of the proton scale

Note that the Einstein equations in the form (3) are universal, i.e. describe the metric in any scale, because their solution depends only on the initial conditions. Choosing these conditions across the proton, we create model that describes the metric of hadrons. Let us consider the metric of the proton, and other elementary particles. In [12] obtained all the solutions of the Yang-Mills equations in the case of a centrally symmetric metric. A particular case of the centrally symmetric metric is

$$y = h_{ij} w^i w^j = -dt^2 + e^{2n} dr^2 + dq^2 + s^2(q) dj^2$$
$$\frac{d^2 s}{dq^2} = -ks \qquad (6)$$

Here $k = const$ is the Gaussian curvature of the quadratic form $dq^2 + s^2(q)dj^2$, function $n = n(r,t)$ is determined by solution of the Yang-Mills equations. As we address the fundamental geometric structure of the observable universe, then we are interested, first of all, the periodic solutions, which form a lattice. This requirement stems from the obvious fact that all points of space should be equal to each other, but each observer can reproduce all the observed variety of phenomena. This is only possible if the basis of the space is a periodic structure.

Among all the solutions of the Yang-Mills equations, obtained in [12] in the case of the metric (6), there is one, which is expressed in terms of Weierstrass elliptic function. In this case, the Yang-Mills equations simplified to the form:

$$
\begin{aligned}
&A_{tt} = \frac{1}{2}(A^2 - k^2), e^n = A_t, \quad t = t \pm r + t_0 \\
&A = \sqrt[3]{12}\wp(t / \sqrt[3]{12}; g_2, g_3), \qquad (7) \\
&b_{11} = -b_{22} = \frac{1}{3}A - \frac{k}{6}, b_{33} = b_{44} = \frac{1}{6}A - \frac{k}{3}, b_{12} = b_{21} = 0.
\end{aligned}
$$

Here g_2, g_3 are the invariants of the Weierstrass function, and $g_2 = k^2\sqrt[3]{12}$, t_0 is a free parameter related to the choice of origin. Metric corresponding to this solution, apparently, can describe the fundamental structure of the universe and, in particular, the structure of elementary particles. The solution of (7) and the corresponding metrics are characterized by two periods, which, obviously, should be related to the parameters of elementary particles.

Note that the sum of the diagonal components of the energy-momentum tensor in this case is $T_{ii} = 2A$. Therefore, setting the characteristic density of matter and space-time scale, we can obtain the general solution, which describes the metric at any scale. We show that the metric (5) described by equations (7) as well. To do this, we consider the solution of the first equation (7) near constant level of density of matter, given by Eq. $A^2 = k^2$.

Introduce a new function and variables according to the formulas

$$f = A - k, t = t + r$$

Suppose that $f^2 << k^2$, then the equations of the model (7) and the solution are of the form

$$
\begin{aligned}
&f_{tt} = kf, \; e^n = f_t \\
&f = f_0 \exp(\sqrt{k}\,t) + f_1 \exp(-\sqrt{k}\,t), \\
&e^n = f_0\sqrt{k}\,e^{\sqrt{k}\,t} - f_1\sqrt{k}\,e^{-\sqrt{k}t} \xrightarrow[t\to\infty]{} f_0\sqrt{k}\,e^{\sqrt{k}\,t}
\end{aligned}
\qquad (8)
$$

In general, as it is known, the corresponding metric is reduced to [12, 14]

$$
y = -dt^2 + ch^2(\sqrt{k}\,t + t_0)dr^2 + dq^2 + \cos^2(\sqrt{k}q + q_0)dj^2 \qquad (9)
$$

Consider the universe at the moment of its existence in a small neighborhood of the coordinate system fixed to the solar system, where $t \approx t$. It is known that the observed average density of matter in the universe is a small quantity compared to the baryon density, so $\sqrt{k} << 1$. Obviously, under these conditions, to match the metrics (5) and (9) it will be enough to put $k = l^2$.

We thus proved that the metric of the observable universe is associated with a metric of the periodic lattice, given by the Weierstrass function (7). Note that although the metric (5) and (9) are similar, but the tensors b_{ij} are not similar. Indeed, the metric (5) is compatible with zero electromagnetic field that describes the components of the tensor $b_{12} = M / bc, \quad b_{34} = N / bc$. Whereas in the metric (7) electromagnetic field is zero, since $b_{12} = 0$.

Therefore, it is necessary to explain the mechanism of the electromagnetic field in the initial lattice (7), which does not contain the electromagnetic field. For this we note that the Yang-Mills field in the linear case is divided into a set of independent electromagnetic fields [15]. Consequently, the electromagnetic field generated at a low energy density, starting from the atomic nuclei and atoms. For its occurrence does not require any additional sources other than the Yang-Mills field and the lattes. In this case the electromagnetic fields have a wavelength multiple of the lattes scale. These waves travel at a constant speed (light) from any source along the lattes in accordance with Maxwell's equations. Thus, the quantization of the electromagnetic waves is a consequence of the presence of the lattice in the metric (7), the period of which is evident in all of the observed phenomena.

Metric model of elementary particles

Suppose that $g_2 = \sqrt[3]{12},\ g_3 = 1$, then the half-periods of the Weierstrass function defined as $w_1 = 1.33003,\ w_2 = 0.66501 + 1.61260i$. Calculation of half-periods and the construction of appropriate 3D images of the Weierstrass function and its first derivative module carried out using the Wolfram Mathematica 9.0 [16]. Consider the metric lattice is formed at the specified parameters - Fig. 2.

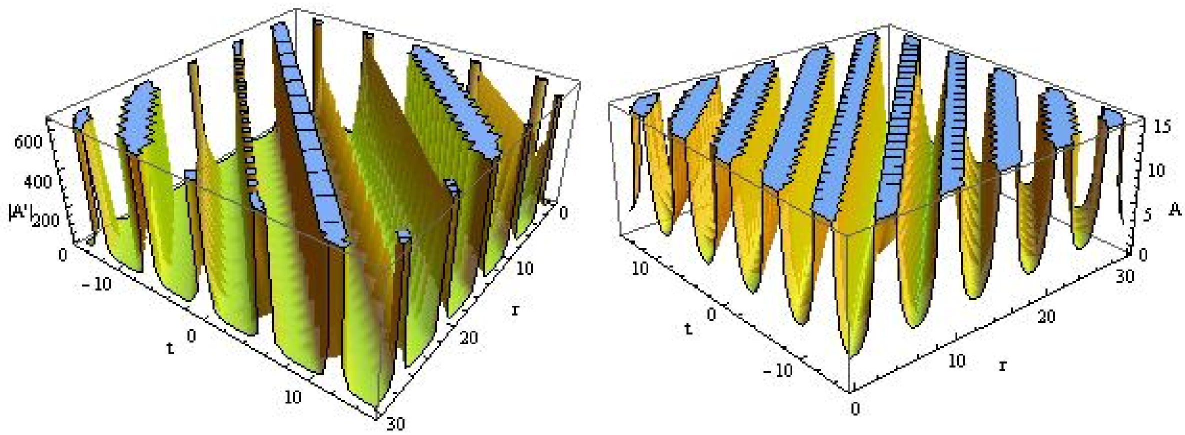

Figure 2: Metric parameters $e^n = |A_t|$ and $T_{ii}/2 = A$ in the case $g_2 = \sqrt[3]{12},\ g_3 = 1$.

As follows from Fig. 2 data, the peaks of the Weierstrass function fuse to form solid walls, stretching along the lines $t \pm r = const$ of the period $2w_1\sqrt[3]{12} \approx 6.09$. These characteristics describe convergent (sign +) or diverging (minus sign) spherical waves. It is easy to see that the solutions of the first equation (7) are symmetric with respect to the change $t \to -t$, however, the metric (5) is asymmetric with respect to time reversal. Therefore, although the microscopic events are reversible in time, macroscopic events are known to be irreversible, which is due, in particular, increasing of entropy. It can be assumed that the emission of spherical waves is the main process in the expanding universe, and then the determination of the metric lattes should assume that $t = t - r + t_0$.

From classical electrodynamics it is known that in the process of electromagnetic wave generation an electrical charge acts as source. In this model, it is most natural to assume that the electromagnetic waves are produced in the interaction of the charge with the lattice. According to present science a charge of hadrons occurs as the sum of fractional charges of quarks. But the origin of the electric charge of the quarks themselves also requires explanation.

In the lattice model can be defined lattice defect type bubble. Lattice with a single bubble is described by the model (7), and the corresponding metric has the form (6). In the bubble we put $A^2 = k^2$, while in the outer region the solution given in the form (7), therefore we have,

$$\begin{aligned} &A^2 = k^2,\ e^n = 0, \quad |t| < t_0 \\ &A = \sqrt[3]{12}\wp(t / \sqrt[3]{12}, g_1, g_2),\ e^n = A_t,\ |t| > t_0 \end{aligned} \tag{10}$$

On the borders of the bubble are continuous function A and its first derivative,

$$k = \sqrt[3]{12}\wp(t_0 / \sqrt[3]{12}, g_1, g_2),\ A_t = 0, |t| = t_0 \tag{11}$$

In the particular case of the lattice in Fig. 2, with the invariants given in the form $g_2 = \sqrt[3]{12},\ g_3 = 1$, we find the first zero and the corresponding parameter of the metric $t_0 = 3.0449983, k = 2.1038034$. The corresponding lattice is shown in Fig. 3.

Similarly, the solution is constructed for the other roots of the second equation (10). All of these roots only effect on the size of the bubble, whereas the value k does not change. Bubble moves at the speed of light along the characteristics $t - r = const$ - Fig. 3. To the outside observer, the bubble is still a homogeneous particle with a finite length.

Any bubble can be turned inside out, just reversed inequalities in (10) - Fig. 4.

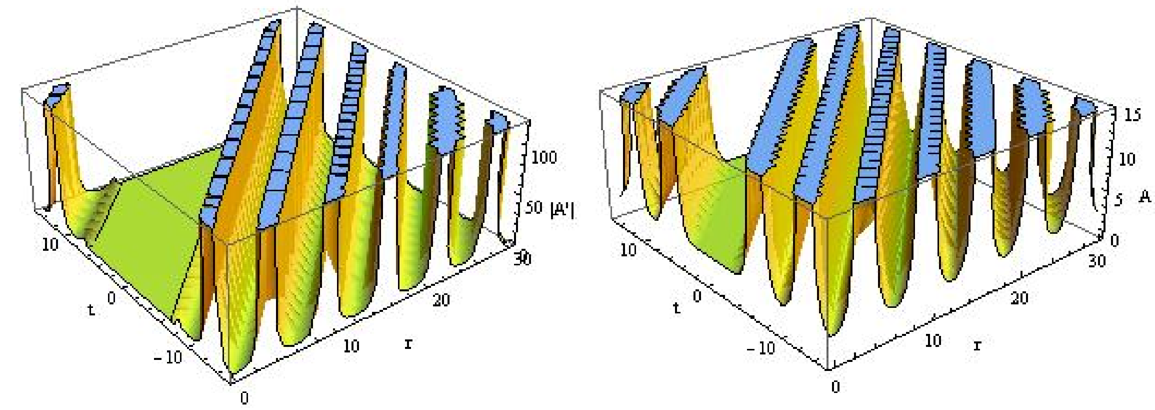

Figure 3: Metric parameters $e^{n} = |A_t|$ and $T_{ii}/2 = A$ in the presence of a homogeneous bubble in the case $g_2 = \sqrt[3]{12},\ g_3 = 1$.

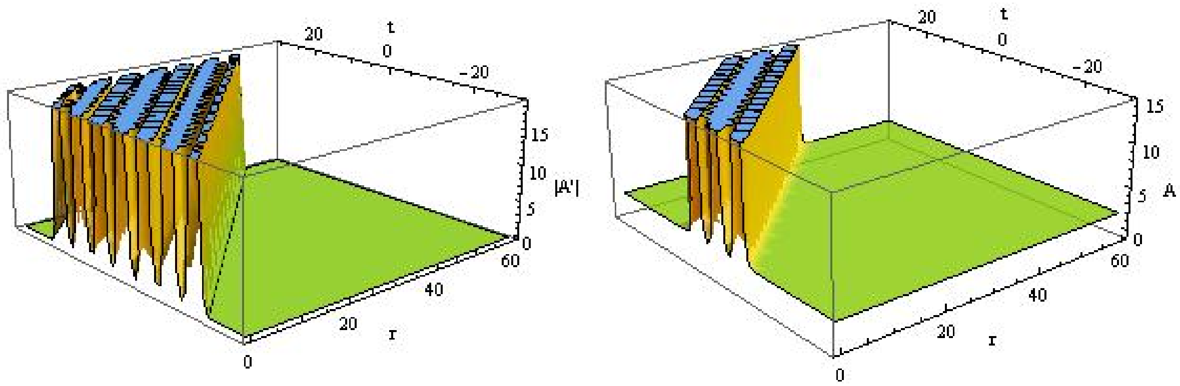

Figure 4: Metric parameters $e^{n} = |A_t|$ and $T_{ii}/2 = A$ in the presence of an inhomogeneous bubble in the case $g_2 = \sqrt[3]{12},\ g_3 = 1$.

In this case, we can extend the definition of the metric in the bubble outer region, using the solution (8), valid for a slight deviation of the metric from the metric bubble, for which the equality $A^2 = k^2$ is true. Then the metric outer space coincides with the metric of the universe, and the metric inside the bubble will be part of the lattice. To the outside observer in our universe this bubble looks like a complex, multi-layered particle with finite scale.

Finally, the third type of particles can be formed as a combination of the first two, plunging the homogeneous bubble of the type shown in Fig. 3 in another bubble, as shown in Fig. 4. The result is a bubble, a restricted shell of finite thickness - Fig. 5. Our next task is to show that such a particle, under certain conditions, can be catching, for example, in a magnetic trap, i.e. it has an electric charge.

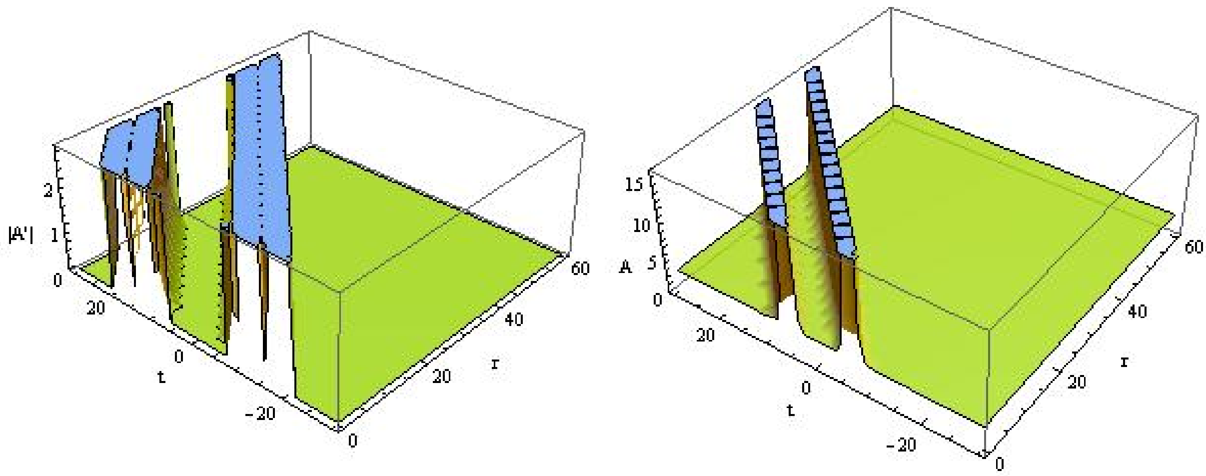

Figure 5: Metric parameters $e^{n} = |A_t|$ and $T_{ii} / 2 = A$ in the presence of a homogeneous bubble in homogeneous outer space in the case $g_2 = \sqrt[3]{12}$, $g_3 = 1$.

First, we note that the particle of the type shown in Fig. 4-5, can be expanded at any rate, not just at the speed of light in our universe. Indeed, the solution build in the inner region depends on a combination of variables $t = t - r$. The default is that the unit of speed is the speed of light. In reality, however, this rate depends on the speed of the external borders, which can be chosen by anyone, including an arbitrarily small. Hence, we find that there may be a spherical particle, which expand in sync with the space of the universe. Therefore, from the point of view of the outside observer they seem having static form like protons.

Second, we note that the cavity inside the particle may contain electromagnetic field, because the parameters in the cavity can be described by equations (2). In the special case of a static electromagnetic field solution of this problem can be written as (5). But the static electromagnetic field in the cavity can be limited only if the cavity is in the external static field or cavity itself contains an electrical charge or charge distribution. It would be logical to attribute these charges to baryonic matter, which is also contained in the cavity, according to (5). Thus, the model developed above allows us to describe the structure of the proton and other hadrons, and to explain the origin of the electric charge.

Finally, we note that the model of particle metrics development above can be combined with the model of the glueball [17], which has been successfully used to predict hadron masses [18].

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