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**МОДЕЛИРОВАНИЕ МАССЫ АДРОНОВ И ЭНЕРГИИ ВОЗБУЖДЕННЫХ СОСТОЯНИЙ АТОМНЫХ ЯДЕР В МОДЕЛИ ГЛЮОННОГО КОНДЕНСАТА****SIMULATION OF HADRON MASSES AND ATOMIC NUCLEI EXCITED STATES IN THE GLUON CONDENSATE MODEL**

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В работе рассмотрена скалярная модель глюонного конденсата, в котором образуются глюболы. Показано, что масса известных адронов и энергия возбужденных состояний ядер описываются с приемлемой точностью интегралом от плотности конденсата по объему глюбола

In this article we consider a scalar model of the gluon condensate, in which bubbles are formed - glue balls. It is shown that the mass of the known hadrons as well as nuclei excited states are described with the acceptable accuracy by the integral of the condensate density in terms of the glueball

**Ключевые слова:** АДРОН, АТОМНОЕ ЯДРО, ВОЗБУЖДЕННЫЕ СОСТОЯНИЯ, ГЛЮОННЫЙ КОНДЕНСАТ, ГЛЮБОЛ, МАССА, СКАЛЯРНОЕ ПОЛЕ

**Keywords:** EXCITED STATES, NUCLEI, GLUON CONDENSATE, GLUEBALL, HADRON, MASS, SCALAR FIELDS

## **Introduction**

According to modern ideas hadrons consist of quarks interacting via vector gauge bosons - gluons. Quantum chromodynamics (QCD), which describes this kind of interaction is extremely complex theory, so the models of elementary particles that are based on QCD, are widely used to simplify and various numerical methods. Glueball is a hypothetical particle predicted by QCD [1]. It is assumed that only consists of glueball gluon condensate. According to the calculations made in the framework of lattice QCD [2], this type of a scalar particle has a mass of about 1730 MeV.

In [3-4] and others have shown that the glueball is the result of the nonlinear interaction of two scalar fields, describing the state of the gluon condensate. In this paper we calculate the hadron masses and energy of the excited states of nuclei based on the model [3-4].

## **Simulation of hadron masses**

In this paper we used a scalar model of the gluon condensate, developed in [3-4]. This model, in the notation of [4] has the form  
<http://ej.kubagro.ru/2012/02/pdf/40.pdf>

$$\begin{aligned} \partial_m \partial^m f &= -f [c^2 + I_1 (f^2 - f_\infty^2)] \\ \partial_m \partial^m c &= -c [f^2 + I_2 (c^2 - c_\infty^2)] \end{aligned} \quad (1)$$

Here  $f, c$  describe the distribution of the scalar field condensate;  $I_1, I_2$  - model parameters;  $f_\infty, c_\infty$  - the eigenvalues of the problem. In the case of spherical symmetry the system (1) is reduced to

$$\begin{aligned} x f'' + 2 f' &= a x f [c^2 + I_1 (f^2 - f_\infty^2)] \\ x c'' + 2 c' &= a x c [f^2 + I_2 (c^2 - c_\infty^2)] \end{aligned} \quad (2)$$

Here we introduced the dimensionless variable  $x = r a^{-1/2}$ . The boundary conditions for the system (2) are:

$$\begin{aligned} f(0) &= 1, \quad f'(0) = 0, \\ c(0) &= c_0, \quad c'(0) = 0. \end{aligned} \quad (3)$$

The system (2) with boundary conditions (3) was solved using Wolfram Mathematica 8 [5] with the values of [4]:

$$a = 1; I_1 = 0.1; I_2 = 1; f_\infty = 1.6171579; c_\infty = 1.49273856 .$$

The results of calculations of functions  $f, c$  are shown in Figure 1. As can be seen from the data shown in Fig. 1 glueball is spherical formation with density dependent on coordinates. In theory [3-4], the density of condensate describes the effective Lagrangian

$$G = -L_{eff} = \langle H_i^A H^{Ai} \rangle - \langle E_i^A E^{Ai} \rangle \quad (4)$$

Here  $E_i^A, H_i^A$  - chromoelectric and chromomagnetic field accordingly.

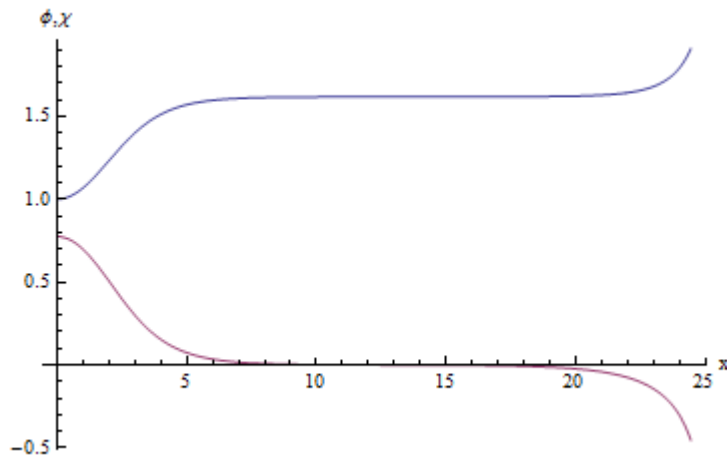


Figure 1: Glueball parameters, calculated according to [4].

Expression of the condensate density as a function of the distribution of the scalar fields is given by [4]

$$G = -\frac{1}{2}(f'^2 + c'^2) + \frac{I_1}{4}(f^2 - f_\infty^2)^2 + \frac{I_2}{4}(c^2 - c_\infty^2)^2 - \frac{I_2}{4}c_\infty^4 - \frac{1}{2}f^2 c^2 \quad (5)$$

In the particular case of the subgroup SU (2) the expression (5) reduces to

$$G_{SU(2)} = -\frac{1}{2}f'^2 + \frac{I_1}{4}(f^2 - f_\infty^2)^2 \quad (6)$$

Expressions (5) - (6) together with the solutions of the problem (2) - (3) were used to simulate the mass of hadrons - Fig. 2-3. Suppose that hadrons consist of a central core - glueball surrounded coat of quark-gluon fields. For each hadron glueball has a certain radius, and the mass of the glueball is determined by the integral of a linear combination of the functions (5) and (6). In addition, the glueball mass contributes surface tension caused by the finite size of the glueball. Thus, the mass is determined according to the glueball

$$m = 4\pi a^{3/2} \int_0^{x_0} (G + bG_{SU(2)} + kr/x)x^2 dx \quad (7)$$

We have considered two models of density  $r = f^2 + c^2$  - Fig. 2,  $r = 1$  -

Fig. 3. Both models have the same accuracy compared with the mass of hadrons, <http://ej.kubagro.ru/2012/02/pdf/40.pdf>

which is apparently due to the behavior of functions  $f, c$  that remain constant over a wide range of variation of the radial coordinate. In addition, separately studied the functional mass of the SU (2) condensate

$$m = 4pa^{3/2} \int_0^{x_0} (G_{SU(2)} + kr/x)x^2 dx \quad (8)$$

Model (7) - (8) has been verified for the entire set of hadrons - Fig. 2-3. Assume that the mass of an individual hadron is proportional to the mass of it glueball therefore have

$$m_H = Hm \quad (9)$$

By changing the parameters of the model, we can achieve matching dependencies (7) - (8) with tabular data hadron masses. To solve this problem, we used the built-in Wolfram Mathematica 8 [5] table of elementary particles with the parameters ParticleData ["Hadron", "Mass"]. The table is extracted data sheet, which adds a number of zero-particles - 175 for the model (7) and 100 for the model (8). These data allow us to combine the origin, in which the mass of a hadron and glueball mass is linear as it proposed in Eq. (9). Data for hadrons are normalized to the maximum element -  $m_Y = 11\,019$  MeV. Next is fitting the model parameters  $a, b, h, k$  - for the model (7), and  $a, h, k$  for the model (8). The parameters  $I_1 = 0.1; I_2 = 1; f_\infty = 1.6171579; c_\infty = 1.49273856$  are stored in all the glueball calculations

This resulted in the following values of the model parameters (7):

$$\begin{aligned} m_H / m_Y &= hm / 4p, \\ r = f^2 + c^2 : a &= 0.0003815; b = 1.792; h = 0.3665, k = 0.0237; \\ r = 1 : a &= 0.0003815; b = 1.792; h = 0.3665; k = 0.061 \end{aligned} \quad (10)$$

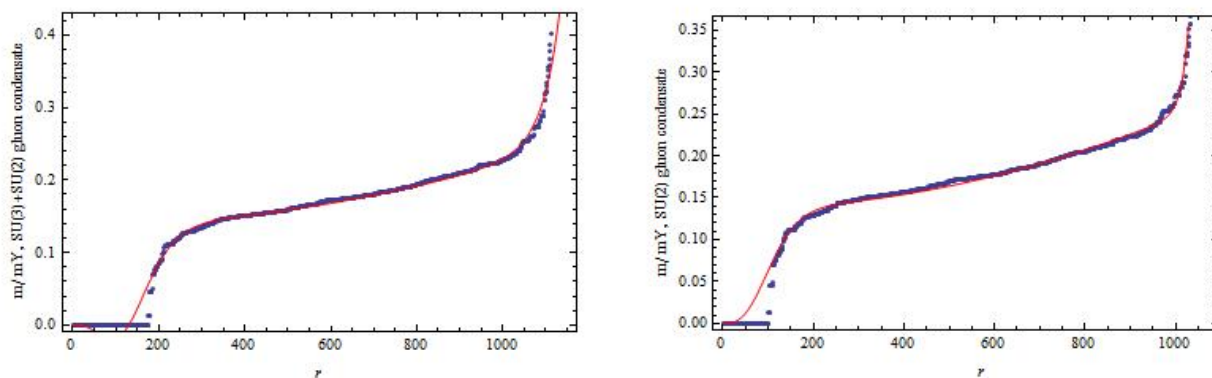


Figure 2: Comparison of the hadron masses with the glueball mass calculated from equations (7) - (8) with  $r = f^2 + c^2$ . Parameters of the model (7):  $a = 0.0003815$ ;  $b = 1.792$ ;  $k = 0.0237$ ;  $h = 0.3665$ . Parameters of the model (8):  $a = 0.000536$ ;  $k = 0.0164$ ;  $h = 0.414$ ;  $m_Y = 11019$  MeV.

Comparison of hadron masses with glueball mass calculated by the model (7) with data (10) is shown in Fig. 2-3. A satisfactory agreement between the calculated and experimental data begins with mass  $r$  - meson of 775.5 MeV and ends at the mass  $Y$  - meson of 4421 MeV. For hadrons smaller and larger mass the linear model (9) is not satisfied.

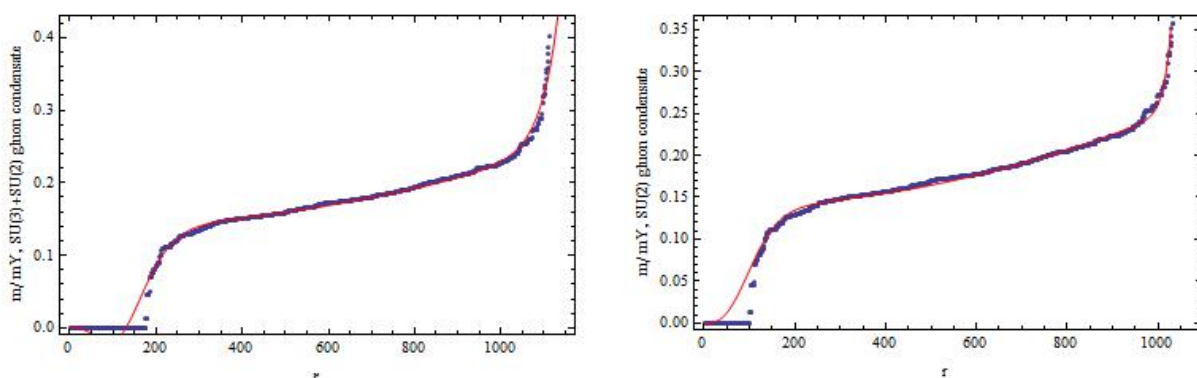


Figure 3: Comparison the hadron masses with the mass of glueball calculated from equations (7) - (8) with  $r = 1$ . Parameters of the model (7):  $a =$

0.0003815;  $b = 1.792$ ;  $k = 0.061$ ;  $h = 0.3665$ . Parameters of the model (8):  $a = 0.000536$ ;  $k = 0.042$ ;  $h = 0.414$ ;  $m_Y = 11019$  MeV.

For the model (8) we obtained the following parameters

$$\begin{aligned} m_H / m_Y &= hm / 4p, \\ r &= f^2 + c^2 : a = 0.000536; h = 0.414, k = 0.0164; \\ r = 1 : & a = 0.000536; h = 0.414; k = 0.042 \end{aligned} \tag{11}$$

Note that the difference in the accuracy of the description of the experimental data between the models (7) and (8) is nominal, but the model (8) contains one less parameter. On the other hand, the difference in density models used to simulate the surface energy is also nominal and limited to a redefinition of the parameter  $k$ , while maintaining the values of other parameters of the model, as it follows from the expressions (10) - (11).

Consider the difference between the theoretical curve and the experimental data in the case of SU (2) condensate - Fig. 4. Here we normalize the mass of hadrons on the mass of proton. Data for the deviation from the theoretical curve given in absolute units, show that the contribution of the orbital motion of the quarks in the hadron mass is not more than 0.1 of the proton mass for light particles and a maximum of 0.15 of the proton mass for the heavy particles. Consequently, we can construct a perturbation theory, using as the main solution glueball and a perturbed motion – orbital motion of quarks.

Thus, we have shown that the linear model (9), which relates the mass of hadrons with a mass of the central core - glueball is performed for a large part of the hadrons, whose mass is in the range from 775.5 MeV to 4421 MeV - about 922 particles from a total of 973. This is evidence in favor of a model of the structure of elementary particles, in which, it is assumed that hadrons contain glueball central core and surrounding fields of quarks and gluons.

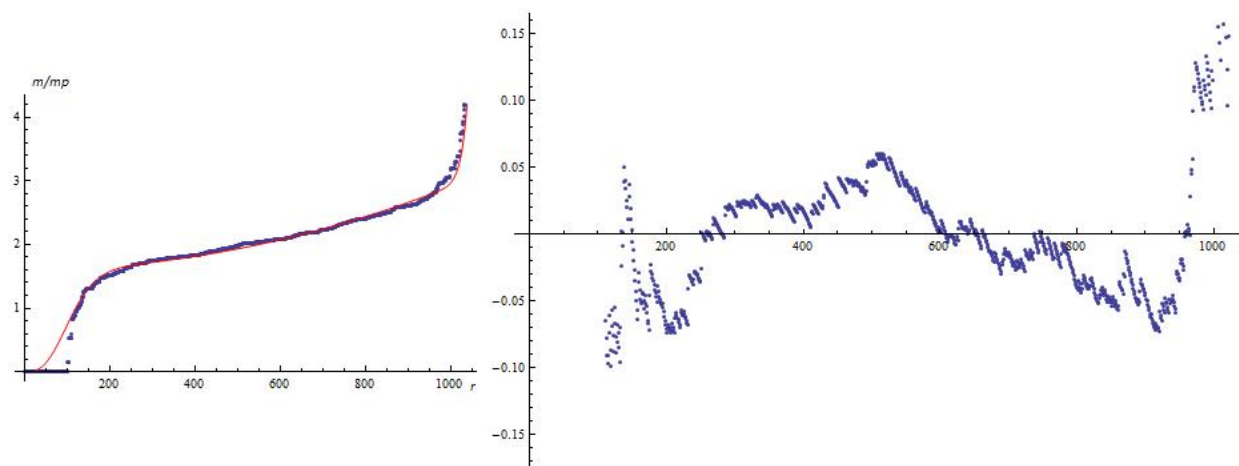


Figure 4: Deviation of hadron masses from the theoretical curve in the case of SU (2) condensate.

### **Modeling the energy of the excited states of nuclei**

According to modern concepts atomic nuclei consist of nucleons - protons and neutrons, which in turn are composed of quarks interacting via vector gauge bosons - gluons. To model the energy of the excited states of nuclei - Fig. 5-6, the model (7) - (9) and a built-in Wolfram Mathematica 8 [5] table of isotopes and associated parameters are used. For example, data on the left of Fig. 5 table of isotopes invoked as `IsotopeData ["Ni58", "ExcitedStateEnergies"]`.

From the table of isotopes extracted data sheet, which adds a number of null states. These data allow us to combine the origin, in which the energy of the excited state and the glueball mass is linear as it proposed in Eq. (9). The data for the energy of the excited states are normalized to the maximum element. Next is fitting the model parameters  $a, b, h, k$  - for the model (7), and  $a, h, k$  for the model (8).

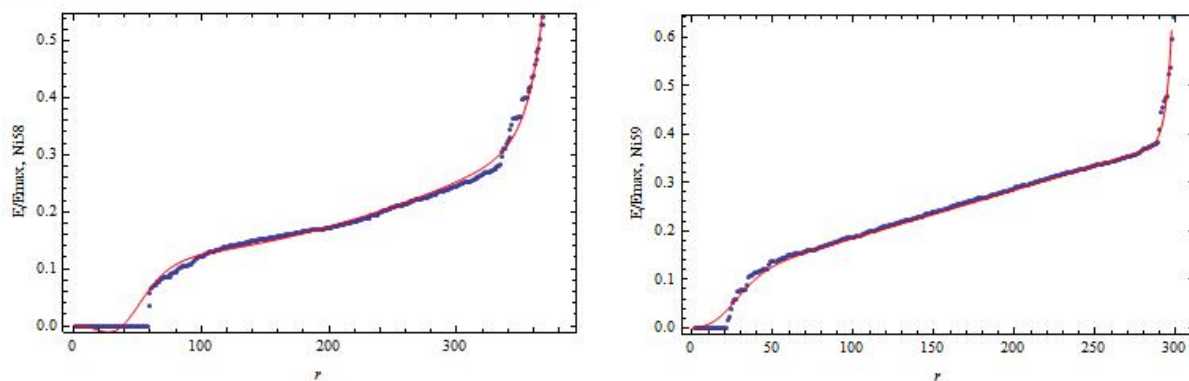


Figure 5: A comparison of the energy of the excited states of isotopes of nickel and glueball energy, calculated from equations (7) - (8) with  $r = f^2 + c^2$ . Parameters of the model (7) for the isotope Ni58:  $k = 0.01906$ ;  $h = 0.2698$ ;  $a = 0.003756$ ;  $b = 1.94$ . Parameters of the model (8) for the isotope Ni59:  $a = 0.0068$ ;  $k = 2.09$ ;  $h = 0.3235$ .

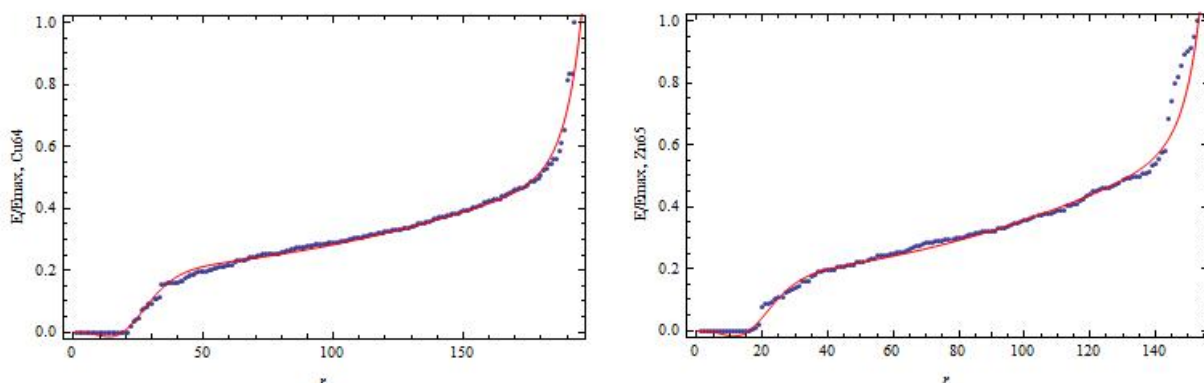


Figure 6: A comparison of the energy of the excited states of isotopes of copper and zinc with energy glueball calculated from equation (7) with. Parameters of the model (7) for the isotope Cu64:  $k = 0.0092$ ;  $h = 0.44$ ;  $a = 0.01356$ ;  $b = 2.085$ . Parameters of the model (7) for the isotope Zn65:  $a = 0.02168$ ;  $b = 1.962$ ;  $k = 0.00984$ ;  $h = 0.44$ .

For each isotope chosen own settings, that indicating there is an individual scenario glueball in each case. For example, for the isotope Ni59 surface tension <http://ej.kubagro.ru/2012/02/pdf/40.pdf>



parameter is not used in the form  $k/x$ , as for isotope Ni58, but in the form  $k/x^2$ , which is apparently due to the influence of angular momentum, which is not considered in the model (1).

Thus, we have shown that the linear model (9), which relates the mass of hadrons with a mass of the central core - glueball, also applies to the excited states of nuclei. In this case the glueball, apparently, should be considered as a bubble, formed in the quantum condensed due to nucleus excitation, just like pores are formed in the solid state and cavitation bubbles in a liquid under tension.

To model the linear stage of the glueball excitation in atomic nuclei one can use, for example, the first equation (1), supplemented by terms that take into account the oscillations of the bubble. Such a model of the quantum harmonic oscillator is widely used in the modeling of nuclear shells [6-7]. In [8-9] for the nuclear shell model used a scalar wave equation in the five-dimensional space, which in a 4-dimensional space is reduced to the first equation (1). In this sense, the model [3-4] (and the model excited states of nuclei developed above) is an obvious non-linear generalization of linear shell model, consistent with the structure of hadrons.

Finally, we note that the glueball is apparently the only one of the possible forms of the organization of hadronic matter, though, for example, Feynman [10] considered it as hadronic "bubbles" and even introduced a special symbol of hadron on Feynman diagrams in the form of bubble. Another possible form is a drop of a Fermi liquid, around which the nucleons in nuclei are concentrated, filling the shell [11]. It is possible that there is a third form, when a drop of quantum liquids is formed as a mixture of two Bose-Einstein condensates [12]. In all these cases in a quantum field there is a central body - bubble or drop, around which is organized the orbital motion of the quarks (in the case of hadrons) or nucleons in atomic nuclei.

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