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ЯДЕРНЫЕ ОБОЛОЧКИ И ПЕРИОДИЧЕСКИЙ ЗАКОН Д.И.МЕНДЕЛЕЕВА. ЧАСТЬ 2.

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На основе теории ядерных взаимодействий и данных по энергии связи нуклонов для всех известных нуклидов установлены параметры, характеризующие периодические закономерности в формировании ядерных оболочек

Ключевые слова: НЕЙТРОН, ПЕРИОДИЧЕСКИЙ ЗАКОН, ПРОТОН, ЭНЕРГИЯ СВЯЗИ, ЯДРО

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NUCLEI SHELLS AND PERIODIC TRENDS. PART 2.

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Parameters describing periodic trends in the formation of nuclear shells have been established based on the theory of nuclear interactions and data on the binding energy of nucleons for the set of known nuclides

Keywords: BINDING ENERGY, PERIODIC TRENDS, PROTON, NEUTRON, NUCLEI

Introduction

The periodic law discovered by Mendeleev in 1869, played a huge role in the development of ideas about the structure of matter. In one of the first formulations of this law states that "the properties of simple bodies, as well as the shape and properties of the compounds of the elements, and therefore the properties of which they form simple and complex bodies are in the periodic table according to their atomic weight" [1]. Using this law, Mendeleev created the periodic table of elements, which is predicted on the basis of new elements - gallium, scandium, germanium, astatine, polonium, technetium, rhenium, and francium.

Ivanenko [2-3], evidently, was one of the first who raised the issue of expansion of the periodic law to include the periodic patterns are observed in atomic nuclei and some exotic formations, such as exotic atoms. According to the theory of nuclear shells [4-5], periodic patterns in the nuclei are explained by analogy with the electron shells, the Pauli principle, which is applied separately for protons and neutrons fill the nuclear envelope. With this expansion of the periodic law of its original wording seems quite logical, since the properties of the nuclei depend not only on the number of protons but also on the number of neutrons.

Thus, the properties of nuclei and the properties of atoms of chemical elements due to the same type on the basis of quantum mechanics and the Pauli principle.

At present, the binding energy of the nucleon measured with high accuracy for almost all known nuclides. However, in contrast to the ionization energy of the atoms, the dependence of the binding energy of the number of nucleons does not contain any explicit reference to the existence of nuclear shells. However, attempts to establish the presence of known nuclear shells and the so-called magic numbers for the deviation from the standard binding energy trend of the Weiszäcker's semi-empirical model [6] and other models [7]. In the present work we investigated the dependence of the energy of the nucleons for all known nuclides with the use of Wolfram Mathematica 8 [8] and three models of the binding energy:

1) Semi-empirical model;

2) 5D model of nuclear interactions [9-11];

3) Information model [12].

The parameters of all models were determined locally for the isotopes of each element. Found that in all three cases, model parameters have a similar behavior depending on the number of protons, which indicates the presence of the internal structure of nuclei.

5D model description

You may notice that the periodic law in the original formulation of Mendeleev is local, as relates properties of simple substances with their atomic weight, which at the time when the law was formulated, was determined by weighing in the gravitational field of the earth. Such a correlation properties of substances and their gravitational properties are reasonable. To answer the question about the fundamental causes that lead to a law of periodicity in nature, consider a general model of atomic nuclei and atoms of matter [10-11]. In this model, the properties of matter are determined by the parameters of the metric tensor in 5-

dimensional space, which depend on a combination of charge and gravitational properties of the central core in the form [9]

$$k = 2gM_A^3 c^2 / Q^4, \quad e^2 / k = 2gM_A / c^2 \qquad (1)$$

Here γ, c, Q - the gravitational constant, the speed of light and charge of the nucleus, respectively. About the nature of the charge will be assumed that the source is an electric charge, but it can be screened in various natural fields. The mechanisms of screening and related fields are discussed below. In the case of proton and electron parameters of the metric tensor (1) are presented in Table 1.

Table 1: The metric tensor parameters

	k, 1/m	e	r_{max}, m	r_{min}, m
e-	1,703163E-28	4,799488E-43	5,87E+27	2,81799E-15
p+	1,054395E-18	1,618178E-36	9,48E+17	1,5347E-18

Note that the maximum scale $r_{\max} = 1/k$ in the case of an electron exceeds the size of the observable universe, while for protons this scale is about 100 light-years. The minimum is the scale $r_{\min} = e/k = e^2/mc^2$ corresponds to the classical radius of a charged particle, which in the case of a proton and an electron is commensurate with the scale and the weak nuclear interactions.

It is easy to see that the second parameter of the model (1) directly included in the formula for the Mendeleev's periodic law [1]. Combining the parameters, we find the nuclear charge in the form: $Q = e^{3/2} c^2 / k\sqrt{2g}$. Consequently, the periodic law in its present formulation can also be expressed through the parameters of the metric tensor (1). The metric tensor can be expanded in the vicinity of a massive center of gravity in five-dimensional space in powers of the dimensionless distance to the source, $\tilde{r} = kr$, here $r = \sqrt{x^2 + y^2 + z^2}$. Consider the form of the metric tensor, which arises when holding the first three terms in the expansion of the metric in the case of central force field with the

gravitational potential in the Newton's form. This choice of metric is justified, primarily because of the specified building the superposition principle holds. Suppose $x^1 = ct, x^2 = x, x^3 = y, x^4 = z$, in this notation we have for the square of the interval in the 4-dimensional space:

$$ds^2 = (1 + 2j/c^2)c^2dt^2 - (1 - 2j/c^2)(dx^2 + dy^2 + dz^2)$$
$$j = -\frac{gM}{r} \quad (2)$$

Assuming that $e^2/k = 2gM/c^2$ we arrive at the expression of the interval depending on the parameters of the metric in the five-dimensional space:

$$ds^2 = (1 - e^2/k)c^2dt^2 - (1 + e^2/k)(dx^2 + dy^2 + dz^2) \quad (3)$$

Further, we note that in this case the metric tensor in four dimensions is diagonal with components

$$g_{11} = 1 - e^2/kr; \quad g_{22} = g_{33} = g_{44} = -(1 + e^2/kr) \quad (4)$$

We define the vector potential of the source associated with the center of gravity in the form

$$g_1 = e/kr, \quad \mathbf{g} = g_1\mathbf{u} \quad (5)$$

Here $\mathbf{u}$ is a vector in three dimensional spaces, which we define below. Hence, we find the scalar and vector potential of electromagnetic field

$$j_e = \frac{Q}{r} = \frac{Mc^2}{e}\frac{e}{kr}, \quad \mathbf{A} = j_e\mathbf{u} \quad (6)$$

To describe the motion of matter in the light of its wave properties, we assume that the standard Hamilton-Jacobi equation in the relativistic mechanics and the Klein-Gordon equation in quantum mechanics arise as a consequence of the wave equation in five-dimensional space [9]. This equation can generally be written as:

$$\frac{1}{\sqrt{-G}}\frac{\partial}{\partial x^m}\left(\sqrt{-G}G^{mn}\frac{\partial}{\partial x^n}\Psi\right) = 0 \quad (7)$$

Here Ψ - the wave function describing, according to (7), the scalar field in five-dimensional space; G^{ik} - the contravariant metric tensor,

$$G^{ik} = h^{-1}\begin{pmatrix} l_1 & 0 & 0 & 0 & -g^1 \\ 0 & l_2 & 0 & 0 & -g^2 \\ 0 & 0 & l_2 & 0 & -g^3 \\ 0 & 0 & 0 & l_2 & -g^4 \\ -g^1 & -g^2 & -g^3 & -g^4 & l \end{pmatrix} \quad (8)$$

$l_1 = (1-e^2/kr)^{-1}; \quad l_2 = -(1+e^2/kr)^{-1}$

$g^1 = l_1 g_1,\ g^2 = l_2 g_2,\ g^3 = l_2 g_3,\ g^4 = l_2 g_4$

$l = 1 + l_1 g_1^2 + l_2(g_2^2 + g_3^2 + g_4^2); \quad G = h^5/(ab^3); \quad h = (kr)^2$.

We further note that in the investigated metrics, depending only on the radial coordinate, is true the following relation

$$F^m = h\frac{\partial}{\partial x^m}\left(\sqrt{-G}G^{mn}\right) = h\frac{\partial r}{\partial x^m}\frac{d}{dr}\left(\sqrt{-G}G^{mn}\right) \quad (9)$$

Taking into account the expressions (8) and (9), we write the wave equation (7) as

$$\frac{l_1}{c^2}\frac{\partial^2\Psi}{\partial t^2} - |\,l_2\,|\nabla^2\Psi + \lambda\frac{\partial^2\Psi}{\partial\rho^2} - 2g^i\frac{\partial^2\Psi}{\partial x^i\partial\rho} + F^\mu\frac{\partial\Psi}{\partial x^\mu} = 0 \quad (10)$$

Note that the last term in equation (10) is of the order $h^2k = k^5r^4 << 1$. Consequently, this term can be dropped in the problems, the characteristic scale which is considerably less than the maximum scale in Table 1. Equation (10) is remarkable in that it does not contain any parameters that characterize the scalar field. The field acquires a mass and charge, not only electric, but also strong in the process of interaction with the central body, which is due only to the metric of 5-dimensional space [9-11].

Consider the problem of the motion of matter around the charged center of gravity, which has an electrical charge and strong, for example, around the proton. In the process of solving this problem is necessary to define the inertial mass of

matter and energy ties. Since equation (10) is linear and homogeneous, this problem can be solved in general.

We introduce a polar coordinate system (r, f, z) with the z axis is directed along the vector potential (8), we put in equation (10)

$$\Psi = y(r)\exp(ilf + ik_z z - iwt - ik_r r) \quad (11)$$

Separating the variables, we find that the radial distribution of matter is described by the following equation (here we dropped, because of its smallness, the last term in equation (10)):

$$-\frac{l_1 w^2}{c^2}y - |l_2|\left(y_{rr} + \frac{1}{r}y_r - \frac{l^2}{r^2}y - k_z^2 y\right) - \lambda k_r^2 y + 2g^1 c^{-1} w k_r y - 2g^z k_z k_r y = 0 \quad (12)$$

Consider the solutions (12) in the case when one can neglect the influence of gravity, i.e. $l_1 \approx -l_2 \approx 1$ but $l = 1 + g_1^2(1-u^2) \neq 1$. Under these conditions, equation (16) reduces to

$$-\frac{w^2}{c^2}y - \left(y_{rr} + \frac{1}{r}y_r - \frac{l^2}{r^2}y - k_z^2 y\right) - \lambda k_r^2 y + 2g^1 c^{-1} w k_r y - 2g^z k_z k_r y = 0 \quad (13)$$

In general, the solution of equation (13) can be represented in the form of a power series [10-11]

$$y = \frac{\exp(-\tilde{r})}{\tilde{r}^a}\sum_{j=0}^{n} c_j \tilde{r}^j \quad (14)$$

It is indicated $\tilde{r} = r / r_n$. Substituting (14) in equation (13), we find

$$\begin{aligned}
&(a^2 - l^2 + k_u)\sum_{j=0}^{n} c_j \tilde{r}^{j-2} + (2a - 1 + k_g r_n)\sum_{j=0}^{n} c_j \tilde{r}^{j-1} + \\
&(1 - k_z^2 r_n^2 + K^2 r_n^2)\sum_{j=0}^{n} c_j \tilde{r}^{j} - \sum_{j=0}^{n} j c_j \tilde{r}^{j-1} - 2a\sum_{j=0}^{n} j c_j \tilde{r}^{j-2} + \\
&\sum_{j=0}^{n} c_j j(j-1)\tilde{r}^{j-2} = 0
\end{aligned} \quad (15)$$

$k_u = (1-u^2)k_r^2 e^2 / k^2$, $K^2 = k_r^2 + w^2/c^2$, $k_g = -2ek_r(k_z u_z + w/c)/k > 0$.

Hence, equating coefficients of like powers, we obtain the equations relating the parameters of the model in the case of excited states

$$a = \sqrt{l^2 - k_u}, \quad r_n = \frac{n+1-2a}{k_g}, \quad \frac{1}{r_n^2} - k_z^2 + k_r^2 + \frac{w^2}{c^2} = 0 \qquad (16)$$

The second equation (16) holds only for values of the exponent, for which the inequality $2a < n+1$ is true. Hence, we find an equation for determining the energy levels

$$\frac{4e^2 k_r^2}{k^2 (n+1-2a)^2}\left(k_z u_z + \frac{w}{c}\right)^2 - k_z^2 + k_r^2 + \frac{w^2}{c^2} = 0 \qquad (17)$$

Equation (17) was used to model the binding energy of nucleons in the nucleus for the entire set of known nuclides [10-11]. In the model [10-11], the core consists of "pure" proton interacting with a scalar field. Part of the "pure" proton is screened by forming N neutrons, as a result there is an atom, consisting of the electron shell and nucleus with electric charge eZ, number of nucleons $A = Z + N$, mass excess $ME = M - A$, and the binding energy

$$E_b = Z(m_e + m_p) + Nm_n - (ME + A \cdot m_u),$$
$$m_u = m(^{12}C)/12 \approx 931.494028 \ MeV$$

Note that we using the standard expression of the mass excess in atomic units. Since two types of charges - scalar and vector, appear in this problem the effect of screening manifests itself not only with respect to the scalar charge (which leads to the formation of neutrons), but also in terms of the vector of the charge, which leads to the formation of the nucleons.

It should be noted that the original metric in the five-dimensional space defined by the metric tensor, which depends only on the parameters of the central

body, i.e. of the total charge and total mass of the nucleons. Different shells can be formed depending on the combination of the charge and mass of the nucleus:

1) Nucleon shell, in which all charges are screened, therefore $e/k == A^2e^2/Am_pc^2 = Ae^2/m_pc^2$;

2) Neutron shell, in which we have $e/k = Ne^2/m_pc^2$;

3) Proton shell, in which $e/k = Ze^2/m_pc^2$.

Using the electron mass and Planck's constant, we define the dimensionless parameters of the model in the form

$$a = \frac{e^2}{\mathbf{h}c},\ S = \frac{(\mathbf{h}k_r)^2}{(m_ec)^2},\ P = \frac{\mathbf{h}k_z}{m_ec},\ E = \frac{\mathbf{h}w}{m_ec^2}$$

$$b_{nl}^X = \frac{4X^2(a\,m_e/m_p)^2}{\left(n+1-2\sqrt{l^2-(1-u^2)SX^2(a\,m_e/m_p)^2}\right)^2} \qquad (18)$$

Here $X = A, N, Z$ in the case of the nucleon, neutron and proton shells, respectively.

Solving equation (17) with respect to energy, we find

$$E_{nl}^X = \frac{-Sb_{nl}^X Pu \pm i\sqrt{-(Sb_{nl}^X Pu)^2 + (Sb_{nl}^X+1)(S-P^2+Sb_{nl}^X P^2u^2)}}{(Sb_{nl}^X+1)} \qquad (19)$$

Note that the parameter in the energy equation (19) can be both real and complex values, which correspond to states with finite lifetime. Given that for most nuclides the decay time is large enough quantity; it can be assumed that the imaginary part of the right-hand side of equation (19) is a small value, which corresponds to a small value of the radicand. Hence we find that for these states the following relation between the parameters

$$P^2 = \frac{S(Sb_{nl}^X+1)}{1+Sb_{nl}^X(1-u^2)} \qquad (20)$$

Substituting in the momentum equation (19), we have

$$E_{nl}^{X} = \frac{S^{3/2} b_{nl}^{X} u}{\sqrt{(Sb_{nl}^{X} + 1)(1 + Sb_{nl}^{X}(1 - u^{2}))}} \quad (21)$$

Hence, we find the dependence of the binding energy per nucleon in the ground state

$$E_{0a}^{X} / A = \frac{S^{3/2} b_{0} u X^{2} / A}{\sqrt{(Sb_{0} X^{2} + 1)(1 + Sb_{0} X^{2}(1 - u^{2}))}} \quad (22)$$

It is indicated $b_0 = (2a\, m_e / m_p (1 - 2a))^2$. Thus, we have established a link the energy of the state parameters with the interaction parameter. Note that the energy of ground state (22) depends on the magnitude of the vector charge, which appears in equations (5) - (6). In [11] have shown that this shows the difference between the interaction of nucleons in nuclei, where the parameter $u \neq 0$, and the interaction between electrons and atomic nuclei, in which $u = 0$.

Equation (22) allows us to describe the dependence of the binding energy of the number of nucleons for all nuclides. The computational model is constructed as follows. Suppose that, based on equation (22) was able to accurately determine the binding energy of one of the isotopes of an element. Without loss of generality we can assume that this is isotope, which contains the minimum number of neutrons. Then the binding energy of all other isotopes of element is defined by

$$\frac{E(N,Z)}{A} = \frac{E_{0a}^{A}(N_{\min}, Z)}{Z + N_{\min}} + \frac{E_{0a}^{N}(N, Z)}{Z + N} - \frac{E_{0a}^{N}(N_{\min}, Z)}{Z + N_{\min}} \quad (23)$$

Model (23) contains the arbitrary choice of the interaction parameter Sb_0. Further, without loss of generality we assume that $Sb_0 = 1$, therefore a momentum scale in the fifth dimension appearing in equations (11) - (17) is established.

Computation of the binding energy of nucleons

We consider three models of the binding energy of nucleons. Standard Weiszäcker semi-empirical model has the form [13]:

$$E_b = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_A (N - Z)^2 A^{-1} + a_5 A^{-3/4} \quad (24)$$

The first term on the right side of (24) describes the increase in binding energy due to the increase in the volume of the system, the second term is due to the contribution of surface energy, the third term describes the contribution of the electric charge of protons, the fourth term due to the contribution of the Fermi energy of nucleons, and finally, the fifth term describes the pairing energy. Since the model (24) depends on five parameters, and model (23), only three, we fix two parameters in equation (24). First, we assume $a_s = 17.23$ that is consistent with the known data [6]. Second, we assume $a_5 = 0$ that due to the specifics of the problem, in which the model parameters are defined locally for a given value of the nuclear charge, and in this case there is no sufficient data to determine this parameter. Consequently, it is necessary to determine the three parameters of the model a_v, a_c, a_A, depending on the number of protons Z.

5D model of the binding energy (23) depends on three parameters. For a given number of protons can be represented as

$$E_b / A = a + \frac{bN^2 / A}{\sqrt{(N^2+1)(1+k(gN)^2)}} \quad (25)$$

The problem is to find the values of model parameters (25) a, b, g, depending on the number of protons Z.

The information model is based on the binding energy changes in terms of energy of a thermodynamic system

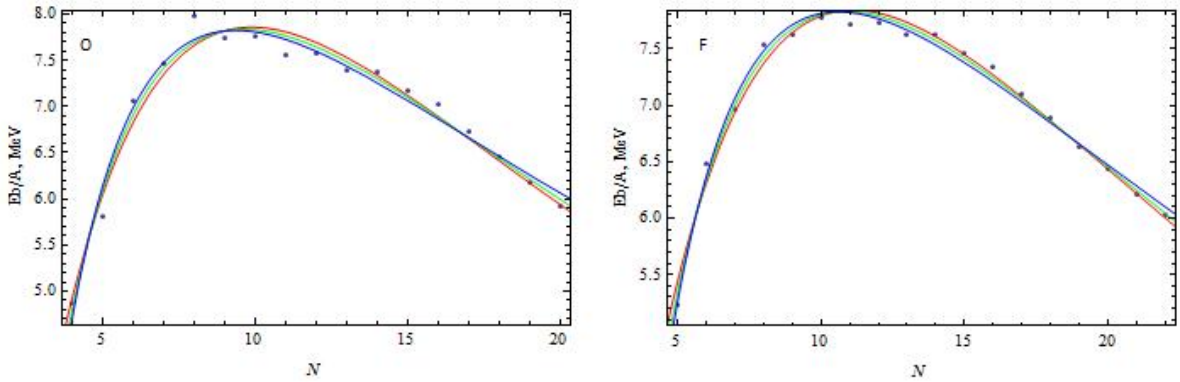
$$dE = TdS - PdV$$

We can assume that in the case of core contributions of pressure and volume is described by the first term on the right side of equation (24), and entropy of the system varies with the number of neutrons, like the entropy of a discrete set [12], thus

$$E_b / A = a_1 + b_1(N / A)(-\ln(N / A) + g_1) \quad (26)$$

Note that in the system Mathematica 8 [8] has built a database of isotopes IsotopeData [], and the procedure for finding the parameters of linear and nonlinear models - Fit, FindFit, NonlinearModelFit. The coefficients of the three models (24) - (26) were calculated in the system [8] for the isotopes of chemical elements (Appendix B shows an example for the model (24)). Model (25) is rigid, so in the calculation of its parameters is introduced numerical coefficient k, which provides the convergence of the solution (see example below in Appendix section). Model (26) can be used without change, but in some cases, a sign in front of the logarithm should be replaced with the opposite sign on the initial iteration. Three models can be compared with experimental data (the corresponding code shows in the Appendix). Results comparing the three models are shown in Figure 1 for isotopes of O, F, Fe, Ni, Pt, Au curves of different colors - green, red and blue for models (24), (25), (26), respectively.

Parameters of the three models, calculated for the number of protons from 4 to 94 are summarized in Table 2.



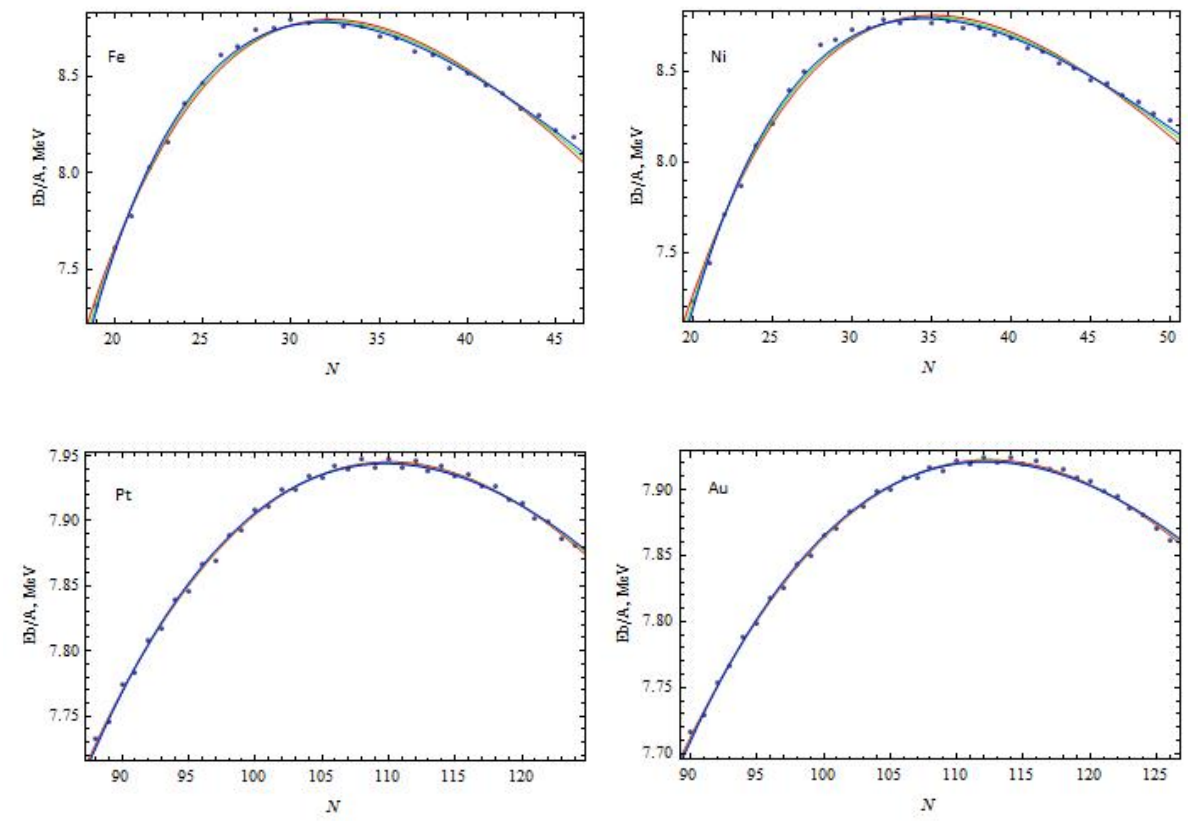


Figure 1: Comparison of three models for the isotopes O, F, Fe, Ni, Pt, Au: the green lines – Weiszäcker's model (24), red lines - 5D model (25), blue lines - an information model.

Table 2: The calculated parameters of models (24) (25) and (26).

Z	a	b	g	a1	b1	g1	av	ac	aA
4	-3.62481	27.327	0.0422609	-20.4234	51.3442	0.355978	14.7091	-0.449354	15.4203
5	-2.52089	23.6371	0.0247448	-14.204	39.0915	0.361751	13.8749	-0.548336	12.7719
6	-2.68681	25.3876	0.0165681	-19.5433	50.4696	0.366144	14.4986	-0.1836	14.5216
7	-3.85908	28.2342	0.0114133	-26.5208	63.2546	0.378508	14.8815	0.262532	16.6353
8	-3.79443	28.7792	0.0086387	-29.0956	68.485	0.382158	15.1319	0.377706	17.331
9	-4.02073	29.1897	0.0067492	-31.347	72.4102	0.38568	15.0451	0.461668	17.8339
10	-3.90832	29.5351	0.0055812	-32.327	74.7037	0.384548	15.066	0.447392	18.1195
11	-4.34471	30.6186	0.0045588	-34.9852	79.4571	0.387794	15.1316	0.538501	18.9135
12	-4.75363	32.0398	0.0037947	-34.9742	79.891	0.386432	15.2473	0.556375	19.3312
13	-4.49257	31.4826	0.0032596	-34.4678	78.9237	0.387226	15.0955	0.520236	19.0377

14	-4.86299	32.9161	0.0028544	-33.7932	78.1938	0.38417	15.1093	0.489994	19.3225
15	-4.92823	32.9845	0.0024462	-34.6704	79.514	0.38844	15.1402	0.532441	19.4656
16	-4.57793	32.2025	0.0021184	-34.6231	79.3658	0.391189	15.1662	0.53731	19.208
17	-4.4887	31.7784	0.0018284	-34.6721	79.0368	0.396352	15.1746	0.568718	19.024
18	-4.20688	31.1873	0.0016155	-34.0293	77.7887	0.398887	15.1715	0.559632	18.6967
19	-4.56505	32.023	0.0014452	-35.2885	79.9604	0.400381	15.1961	0.58579	19.1674
20	-4.64648	32.4197	0.0013115	-35.6665	80.765	0.400448	15.2171	0.580203	19.3655
21	-5.26814	33.8858	0.00119	-38.196	85.348	0.400375	15.2774	0.617602	20.2984
22	-5.67551	35.0752	0.0010958	-40.1961	89.2201	0.398872	15.3419	0.628125	21.0693
23	-6.05237	35.9337	0.0009984	-41.5921	91.6283	0.400196	15.3843	0.651503	21.5696
24	-6.11985	36.2127	0.0009177	-41.99	92.3902	0.400701	15.4094	0.648387	21.7216
25	-6.24784	36.4664	0.0008418	-42.4793	93.1508	0.401959	15.4076	0.653399	21.8633
26	-6.28165	36.6336	0.0007779	-41.7491	91.9181	0.401584	15.3744	0.633746	21.7385
27	-5.91711	35.5786	0.0007103	-40.9674	90.2611	0.403841	15.3089	0.622372	21.2347
28	-5.78781	35.2243	0.0006501	-39.7398	87.903	0.405936	15.3041	0.614025	20.8196
29	-4.99568	33.1639	0.0006016	-39.4301	87.1016	0.408116	15.1827	0.590204	20.1834
30	-4.57451	32.0833	0.0005569	-38.1885	84.7455	0.409463	15.0929	0.566158	19.6017
31	-4.63175	32.0402	0.0005134	-38.5366	85.1258	0.411775	15.099	0.577839	19.6222
32	-4.63363	32.0235	0.0004795	-38.6924	85.3594	0.412437	15.0928	0.575169	19.6399
33	-4.89913	32.5176	0.0004462	-39.7378	87.0587	0.413959	15.1314	0.593458	19.9647
34	-4.92286	32.5903	0.0004204	-42.1626	91.2954	0.415514	15.2229	0.61443	20.4833
35	-5.32932	33.4376	0.0003932	-43.7459	93.9747	0.416771	15.2993	0.639397	21.0178
36	-5.73261	34.4072	0.0003708	-45.5038	97.1115	0.417121	15.4002	0.659816	21.6395
37	-6.02828	35.0052	0.0003487	-46.3959	98.5568	0.418232	15.4387	0.673052	21.9657
38	-6.30199	35.6495	0.0003294	-47.6848	100.826	0.418783	15.5158	0.686486	22.3972
39	-5.99714	34.8385	0.0003117	-47.4196	100.218	0.419675	15.4344	0.670048	22.0897
40	-5.80949	34.4269	0.0002976	-46.5407	98.6955	0.419209	15.3424	0.644418	21.7953
41	-5.29964	33.138	0.0002831	-45.367	96.4927	0.41975	15.1905	0.61341	21.1702
42	-5.18736	32.8917	0.0002706	-44.7795	95.4757	0.419392	15.1213	0.594908	20.9812
43	-4.72927	31.7071	0.0002573	-43.0526	92.2911	0.419885	14.9618	0.56367	20.276
44	-4.68112	31.5894	0.0002456	-42.7163	91.6774	0.420027	14.925	0.553454	20.1688
45	-4.30091	30.5779	0.0002333	-40.9962	88.4722	0.420954	14.787	0.528102	19.5122
46	-3.98847	29.8081	0.0002226	-39.6453	86.0108	0.421472	14.688	0.506142	19.0082
47	-3.01173	27.3123	0.0002095	-36.5769	80.3064	0.423684	14.4568	0.463584	17.6412
48	-2.82675	26.8101	0.0001988	-35.6621	78.5572	0.425169	14.4103	0.454071	17.2941
49	-2.64241	26.2272	0.0001878	-34.8193	76.8483	0.427347	14.3553	0.447902	16.9268
50	-2.75243	26.3929	0.0001774	-35.0131	77.0089	0.429706	14.4058	0.458403	16.982
51	-3.10147	27.0323	0.0001675	-36.9114	80.0647	0.432777	14.5294	0.489927	17.5059
52	-4.1087	29.2927	0.0001595	-40.8443	86.8587	0.434208	14.8168	0.548111	18.9091
53	-4.88685	30.9368	0.0001512	-44.1034	92.3213	0.436673	15.0542	0.599221	19.986
54	-5.77758	32.9822	0.0001456	-47.6512	98.5393	0.436678	15.2876	0.642985	21.2657
55	-6.49219	34.5295	0.0001395	-50.793	103.932	0.437431	15.4808	0.68223	22.3091
56	-6.55646	34.6463	0.0001344	-50.8494	103.993	0.437599	15.4654	0.677452	22.3435

57	-6.71471	34.8831	0.0001287	-51.7677	105.403	0.439024	15.5096	0.688154	22.5647
58	-6.77462	34.9917	0.0001243	-51.8113	105.447	0.43912	15.4913	0.682866	22.5949
59	-6.61779	34.5162	0.0001194	-50.9097	103.723	0.439992	15.4053	0.669129	22.2552
60	-6.64243	34.5196	0.0001151	-51.3111	104.33	0.440713	15.414	0.669337	22.3221
61	-6.5384	34.155	0.0001105	-50.6191	102.949	0.441853	15.3515	0.660245	22.0511
62	-6.571	34.1944	0.0001069	-50.5155	102.727	0.442029	15.3256	0.654196	22.0297
63	-6.54571	34.0061	0.0001026	-50.1268	101.863	0.44335	15.2915	0.650338	21.8645
64	-6.60085	34.0906	9.923E-05	-51.0797	103.449	0.443885	15.3239	0.654754	22.0742
65	-6.50471	33.7317	9.524E-05	-50.3913	102.047	0.445361	15.272	0.647954	21.799
66	-6.38636	33.4135	9.218E-05	-49.6857	100.758	0.445726	15.2046	0.635322	21.5537
67	-6.25463	32.9776	8.857E-05	-48.8482	99.1038	0.447197	15.1396	0.626537	21.2265
68	-5.86922	32.0605	8.596E-05	-47.5027	96.7233	0.447365	15.0027	0.602623	20.6984
69	-5.54572	31.1761	8.249E-05	-45.8781	93.681	0.449166	14.8826	0.585267	20.0847
70	-5.1355	30.2033	8.013E-05	-44.3569	91.0016	0.449317	14.7324	0.560059	19.5073
71	-5.13124	30.0637	7.707E-05	-44.2486	90.6327	0.450782	14.7158	0.559955	19.4139
72	-5.1398	30.0561	7.505E-05	-44.8237	91.6306	0.45047	14.7088	0.558424	19.5368
73	-5.08636	29.7848	7.199E-05	-44.3017	90.492	0.452533	14.6794	0.556499	19.3111
74	-5.15849	29.8641	6.949E-05	-44.7317	91.095	0.4538	14.7085	0.56144	19.3941
75	-5.18135	29.7528	6.652E-05	-44.5443	90.5021	0.456279	14.7143	0.56513	19.2703
76	-4.81392	28.8112	6.396E-05	-42.837	87.3653	0.458217	14.5937	0.546876	18.628
77	-4.739	28.4772	6.119E-05	-42.4773	86.4801	0.46079	14.5782	0.547303	18.4114
78	-4.3432	27.458	5.861E-05	-40.8502	83.4595	0.463247	14.4649	0.530748	17.7622
79	-4.11711	26.7572	5.566E-05	-40.1393	81.9009	0.46695	14.4327	0.528879	17.367
80	-5.44848	29.7541	5.483E-05	-45.8294	91.7441	0.465145	14.85	0.591188	19.3432
81	-6.3089	31.5529	5.29E-05	-50.0175	98.6963	0.466802	15.1852	0.643356	20.6374
82	-7.35854	33.897	5.192E-05	-54.2297	105.945	0.465925	15.4981	0.688685	22.1181
83	-10.6838	41.2669	5.081E-05	-69.5929	132.156	0.465622	16.6923	0.86676	27.1828
84	-12.0013	44.1892	4.977E-05	-75.7497	142.678	0.465302	17.1429	0.930859	29.2048
85	-14.341	49.3045	4.842E-05	-87.2432	162.142	0.465986	18.0281	1.06011	32.8569
86	-13.1541	46.6637	4.754E-05	-57.8229	112.377	0.461763	17.588	0.991697	31.3175
87	-12.0067	44.0411	4.647E-05	-10.9011	33.4322	0.416778	17.1993	0.933971	29.8626
88	-10.7392	41.227	4.573E-05	-33.0918	70.7649	0.449922	16.6866	0.857065	28.0244
89	-9.3985	38.2087	4.493E-05	-0.240897	15.592	0.333918	16.162	0.781219	26.1047
90	-7.66393	34.3582	4.437E-05	1.39374	12.7593	0.305706	15.468	0.680812	23.5375
91	-6.60717	31.912	4.33E-05	1.25935	12.7706	0.319387	15.048	0.622713	21.8927
92	-5.29195	28.9599	4.261E-05	4.6173	7.29986	0.171406	14.5418	0.55151	19.9757
93	-10.3571	39.5927	3.857E-05	5.70092	5.94733	0.0177703	16.745	0.867284	27.4873
94	-10.6928	40.2478	3.754E-05	6.13997	5.31963	-0.0623392	16.8843	0.884977	27.9953

The Appendix provides the text of programs to calculate and plot the model parameters on the number of protons - Fig. 2-4. The parameters of all three models

vary with the number of protons. Since the 5D model is rigid, it uses the above-introduced coefficient k, which provides the convergence of solutions depending on the number of protons in the form $k = 0.9592/Z^{2.209}$. Shown in Fig. 3 parameter g is calculated with respect to this factor.

Analyzing the data given in Table 2 and Fig. 2-4, we can conclude that there is no universal model that describes the entire set of nuclides. From the data presented in Fig. 1, it follows that all three models describe equally well the binding energy of the isotopes of individual elements. In this sense, the model of Weiszäcker cannot be regarded as a universal model, even with the term describing the pairing energy.

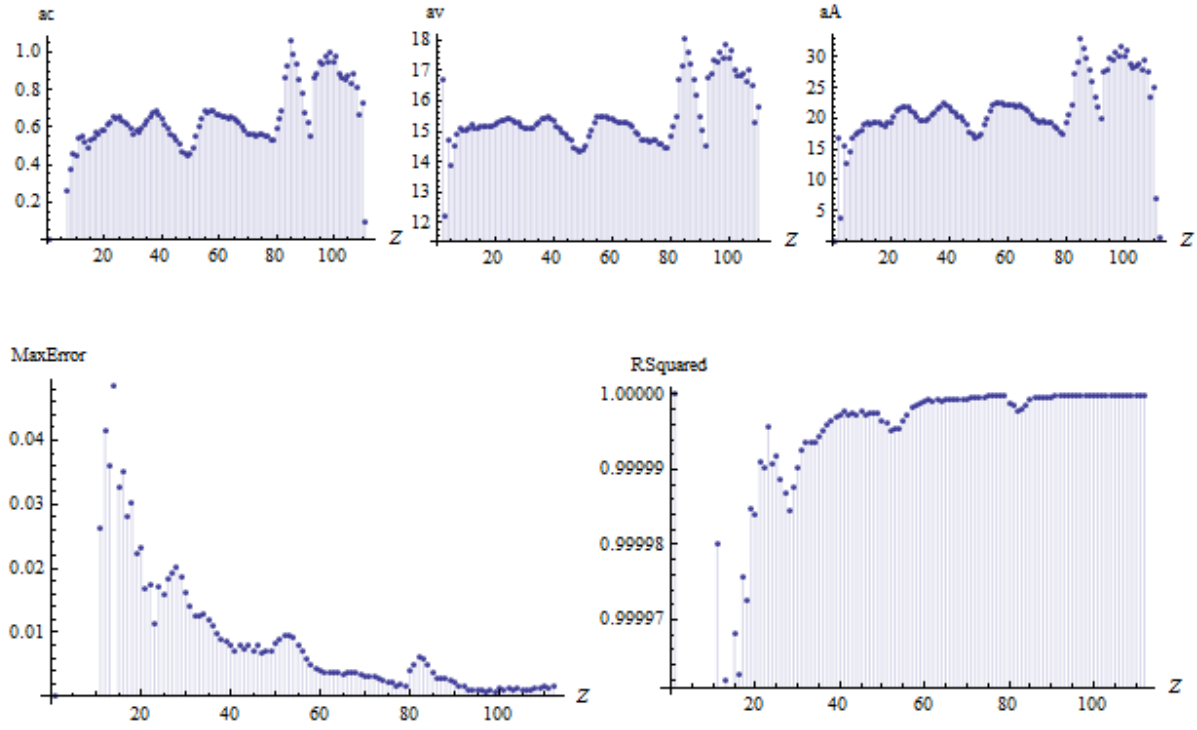


Figure2: The dependence of Weiszäcker model parameters on the number of protons, the lower figures show the value of standard deviation RSquared and maximum absolute prediction error of the binding energy - MaxError.

Note the similarity in the behavior of parameters of three models: the parameters reach extreme values at the same or similar values of Z - Table 3. These results indicate the presence of nuclear structure, but the point of extremes do not coincide with the magic numbers of protons - 2, 8, 20, 28, 50, 82, as defined in the standard nuclear shell model [3-6]. A similar result was obtained in [11], in which the local parameters 5D model depending on the number of neutrons have been calculated. As it turned out, the number of neutrons corresponding to the extreme values of the parameters of the model 5D, close to the magic numbers, but nowhere with them do not match.

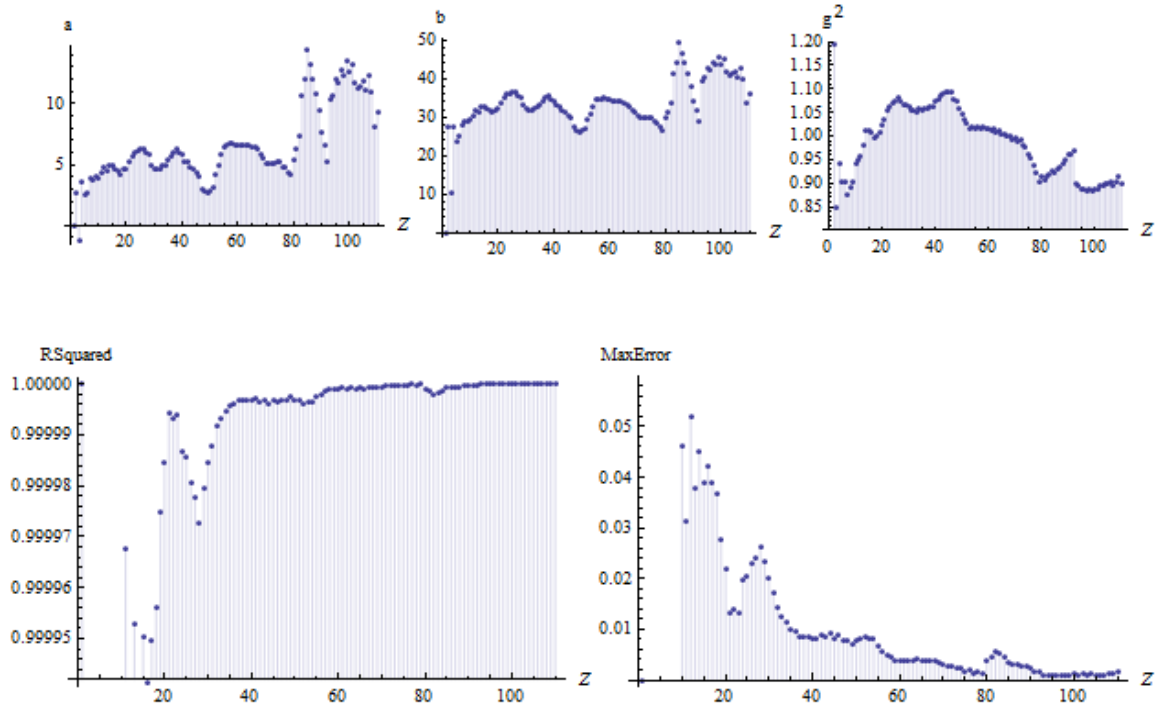


Figure 3: The dependence of the parameters of the 5D model on the number of protons.

In this regard, we note in Table 3 and Fig. 2-4 three points that fall on the elements Z = 31,32,85 - Ga (Gallium), Ge (germanium), At (astatine). Gallium and germanium were predicted by Mendeleev in 1870 and discovered in 1875 and 1885, respectively. Astatine predicted by Mendeleev was artificially synthesized only in 1940. Note three extreme coinciding with the Z = 26, 79, 92 - Fe (iron), Au (gold) and U (uranium). There is no doubt that the iron is clearly identified in nature and has long been used in human practice. The role of gold and uranium in human history cannot be overestimated. It is also interesting that only in the 5D model, the binding energy of one of the extremes have the element with proton number Z = 26 - iron.

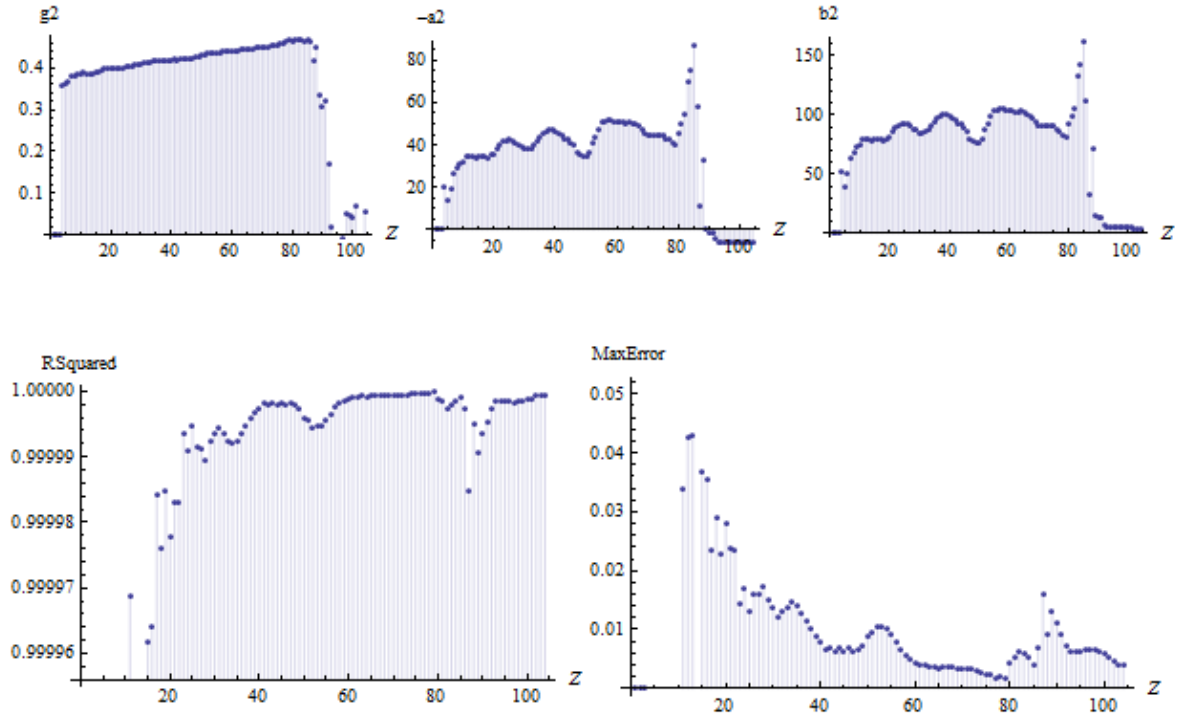


Figure 4: The dependence of the parameters of the information model on the number of protons.

Table 3: Extreme values of the model parameters

Z	a	b	a1	b1	av	aA
5	-2.52089	23.6371	-14.204	39.0915	13.8749	12.7719
18	-4.20688	31.1873	-34.0293	77.7887	15.1715	18.6967
25			-42.4793	93.1508	15.4076	21.8633
26	-6.28165	36.6336				
31	-4.63175		-38.5366	85.1258		19.6222
32		32.0235			15.0928	
38	-6.30199	35.6495	-47.6848	100.826	15.5158	22.3972
49	-2.64241	26.2272	-34.8193	76.8483	14.3553	16.9268
58	-6.77462	34.9917	-51.8113	105.447	15.4913	22.5949
79	-4.11711	26.7572	-40.1393	81.9009	14.4327	17.367
85	-14.341	49.3045	-87.2432	162.142	18.0281	32.8569
92	-5.29195	28.9599	4.6173	7.29986	14.5418	19.9757

We can assume that there is a version of the periodic table, in which periods are associated with the trend shown in Fig. 2-4 and in Table 2. These results suggest that the periodic properties of the nuclei of atomic elements depend on the number of protons (charge), in line with the modern formulation of the periodic law [14]. It has been previously established [11] that the periodic properties of nuclei depend on the number of neutrons, which is reflected in the original formulation of Mendeleev's periodic law. The Appendix gives the texts of programs to calculate the model parameters depending on the number of neutrons - Fig. 5-6.

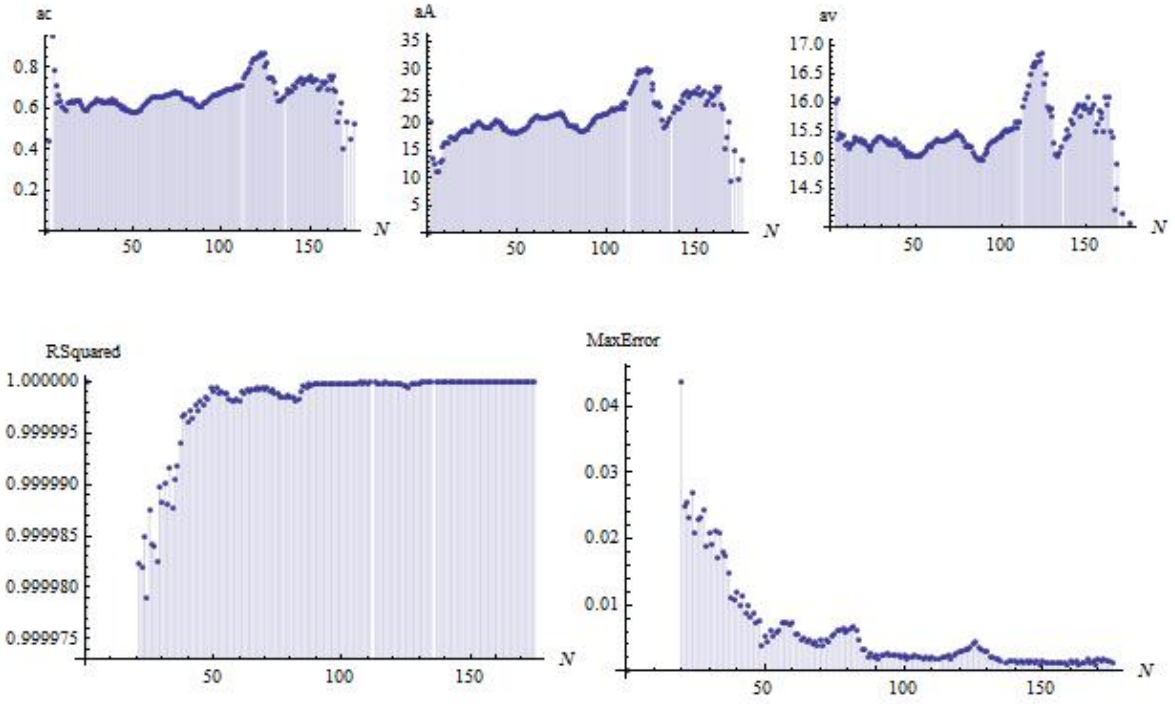


Figure 5: The dependence of Weiszäcker model parameters on the number of neutrons, the lower figures show the value of standard deviation and maximum absolute prediction error of the binding energy.

Model (24) used in this case without change and 5D model takes the form

$$E_b / A = a + \frac{bZ^2 / A}{\sqrt{(Z^2+1)(1+k(gZ)^2)}} \qquad (27)$$

Since the 5D model is rigid, it uses a numerical coefficient k, which provides the convergence of solutions depending on the number of neutrons in the form $k = 0.0025 - 0.0003 \ln N$.

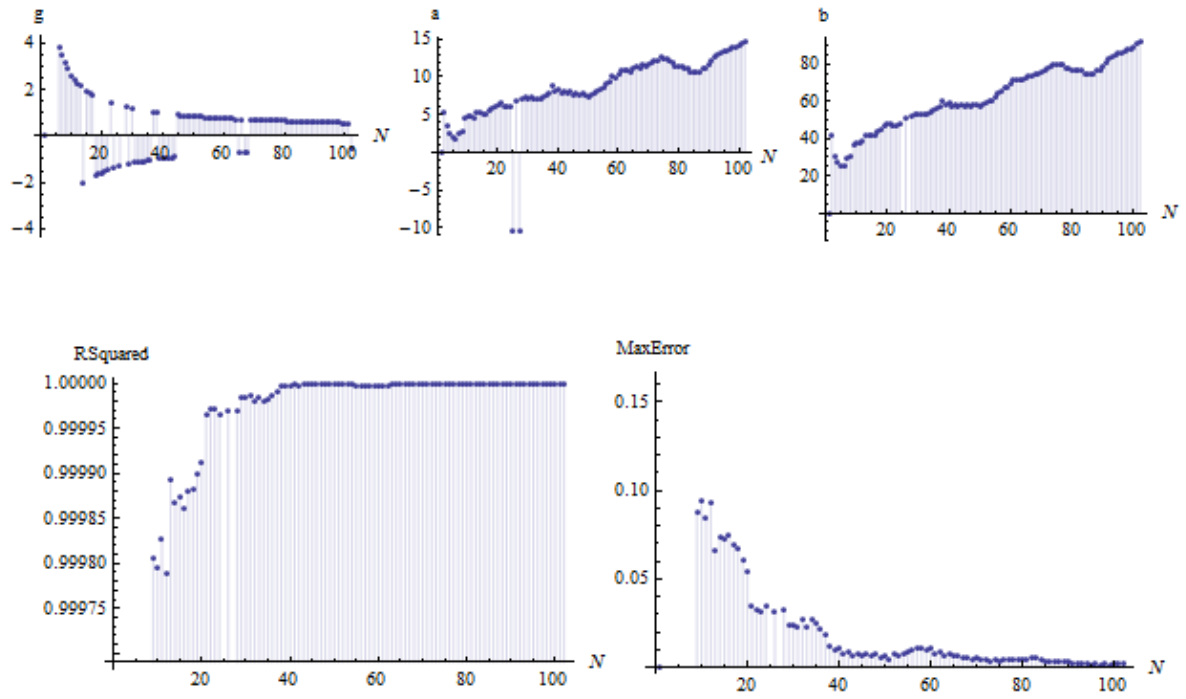


Figure 6: The dependence of the parameters of the 5D model on the number of neutrons.

The data presented in Fig. 5 that the Weiszäcker model parameters depend on the number of neutrons, and these dependencies are not monotonic, which indicates the presence of nuclear structure. Thus, we have shown that the binding energy of all known nuclides can be described approximately with the same accuracy by any of three models (24)-(26). This means that the nucleus can be regarded as a charged liquid drop (Weiszäcker model), and as a set of shielded "clean" protons in the five-dimensional space [10-11], and as a statistical (information) system [12].

Note that the droplet model of the nucleus had a large development in the 30-50s of last century. On the other hand, 5D model is theoretically justified by Kaluza [15], Einstein [16-19], Pauli and Einstein [20], Rumer [21], Dzhunushaliev [22], and in our papers [9-11] as well. The information model of the nucleus also has a

great potential in terms of its expansion, taking into account the spin angular momentum and other quantum numbers, as well as quantum chaos [6-7, 23-24].

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Appendix

Source code for calculating the Weiszäcker model parameters in Table 2:

```
Do[ model = av - 17.23*(Z + x)^(-1/3) + ac*(Z*Z)*(Z + x)^(-4/3) + aA*((x - Z)^2)*(Z + x)^(-2);
 Eb = Table[IsotopeData[#, prop], {prop, {"NeutronNumber", "BindingEnergy"}}] & /@  IsotopeData[Z];
 nlm = FindFit[Eb, model, {{ av, 15.5}, {ac, -.628528}, {aA, -22.03}}, x]; Print[Z, nlm], {Z, 1, 118}]
```

Source code for the comparison of three models - Fig. 1:

```
Z = 78; k = 0.000049;
Eb = Table[IsotopeData[#,  prop], {prop, {"NeutronNumber", "BindingEnergy"}}] & /@  IsotopeData[Z];
nlm = NonlinearModelFit[Eb, a + b*(x*x/(Z*1. + x))*((x^2 + 1)*(1 + k*(g*x)^2))^(-.5) , {a, b,  g}, x];
nlm1 = NonlinearModelFit[Eb,  av - 17.23*(Z + x)^(-1/3) - ac*(Z*Z)*(Z + x)^(-4/3) - aA*((x - Z)^2)*(Z + x)^(-2) , {av, ac, aA}, x];
nlm2 = NonlinearModelFit[Eb,  a2 + b2*(x/(Z*1. + x))*(-Log[x/(Z*1. + x)] + g2) , {a2, b2, g2}, x];
Show[ListPlot[Eb], Plot[{nlm[x], nlm1[x], nlm2[x]}, {x, 1., 180.},  PlotStyle -> {Red, Green, Blue}], Frame -> True,
 FrameLabel -> {N, "Eb/A, MeV"}]
```

Source code for calculating the Weiszäcker model parameters depending on the number of protons (Fig. 2):

```
par = {0.}; para = {0.}; parc = {0.}; RSq = {1.}; MaxEr = {0.};
Do[ Eb = DeleteCases[Table[IsotopeData[#,prop], {prop, {"NeutronNumber", "BindingEnergy"}}] & /@
   IsotopeData[Z], {_, Missing["Unknown"]}];
```

```
nlm = NonlinearModelFit[Eb,  av - 17.23*(Z + x)^(-1/3) - ac*(Z*Z)*(Z + x)^(-4/3) -  aA*((x - Z)^2)*(Z + x)^(-2) , {av, ac, aA}, x];
RSq = {RSq, nlm["RSquared"]} // Flatten;
MaxEr = {MaxEr, Last[Sort[nlm["MeanPredictionErrors"]]]} // Flatten;
para = {para, av /. nlm["BestFitParameters"]} // Flatten;
parc = {parc, ac /. nlm["BestFitParameters"]} // Flatten;
par = {par, aA /. nlm["BestFitParameters"]} // Flatten, {Z, 2, 112}]
ListPlot[par, Filling -> Axis, AxesLabel -> {Z, aA},
 ImageSize -> {200, 200}] ListPlot[para, Filling -> Axis,
 AxesLabel -> {Z, av}, ImageSize -> {200, 200}] ListPlot[parc,
 Filling -> Axis, AxesLabel -> {Z, ac}, ImageSize -> {200, 200}]
ListPlot[RSq, Filling -> Axis, AxesLabel -> {Z, "RSquared"},
 ImageSize -> {300, 300}, DataRange -> Automatic] ListPlot[MaxEr,
 Filling -> Axis, AxesLabel -> {Z, "MaxError"},
 ImageSize -> {300, 300}, DataRange -> Automatic]
```

Source code for calculating the Weiszäcker model parameters depending on the number of neutrons (Fig. 5):

```
par = {.0}; para = {.0}; parc = {.0};
Do[model = av - 17.23*(nn + x)^(-1/3) - ac*(x*x)*(nn + x)^(-4/3) -
   aA*((x - nn)^2)*(x + nn)^(-2) ;
 Eb = Drop[
   Cases[DeleteCases[
     Table[{a - z, z, IsotopeData[{z, a}, "BindingEnergy"]}, {z, 1,
       118}, {a,
       IsotopeData[#, "MassNumber"] & /@ IsotopeData[z]}], {_,
      Missing["Unknown"]}] // Flatten[#, 1] &, {nn, _, _}],
   None, {1}];
 nlm = FindFit[Eb, model, {{ av, 15.5}, {ac, 0.628528}, {aA, 22.03}},
   x]; para = {para, av /. nlm} // Flatten;
 parc = {parc, ac /. nlm} // Flatten;
 par = {par, aA /. nlm} // Flatten, {nn, 2, 175}]
ListPlot[par, Filling -> Axis, AxesLabel -> {N, aA}]
ListPlot[para, Filling -> Axis, AxesLabel -> {N, av}]
ListPlot[parc, Filling -> Axis, AxesLabel -> {N, ac}]
```

Source code for calculating the dependence of 5D model parameters on the number of protons (Fig. 3):

```
par = {0.}; para = {0.}; parc = {0.}; RSq = {1.}; MaxEr = {0.};
Do[ Eb = DeleteCases[Table[IsotopeData[#, prop], {prop, {"NeutronNumber", "BindingEnergy"}}] & /@
    IsotopeData[Z], {_, Missing["Unknown"]}];
 nlm = NonlinearModelFit[Eb,  a + b*(x*x/(Z*1. + x))*((x^2 +1)*(1 + (0.9592/Z^2.209)*(g*x)^2))^(-.5), {a, b, g}, x];
 para = {para, -a /. nlm["BestFitParameters"]} // Flatten;
 RSq = {RSq, nlm["RSquared"]} // Flatten;
```

```
 MaxEr = {MaxEr, Last[Sort[nlm["MeanPredictionErrors"]]]} // Flatten;
 parc = {parc, b /. nlm["BestFitParameters"]} // Flatten;
 par = {par, g^2 /. nlm["BestFitParameters"]} // Flatten, {Z, 2, 110}]
ListPlot[par, Filling -> Axis, AxesLabel -> {Z, "g"}, ImageSize -> {200, 200}, PlotRange -> {0.8, 1.2}] ListPlot[para,
  Filling -> Axis, AxesLabel -> {Z, "a"},  ImageSize -> {200, 200}] ListPlot[parc, Filling -> Axis,
  AxesLabel -> {Z, "b"}, ImageSize -> {200, 200}]
ListPlot[RSq, Filling -> Axis, AxesLabel -> {Z, "RSquared"},  ImageSize -> {200, 200}, DataRange -> Automatic] ListPlot[MaxEr,
  Filling -> Axis, AxesLabel -> {Z, "MaxError"},
  ImageSize -> {200, 200}, DataRange -> Automatic]
```

Source code for calculating the dependence of 5D model parameters on the number of neutrons (Fig. 6):

```
par = {0.};para = {0.};parc = {0.};RSq = {0.};MaxEr = {0.};
Do[ Eb = Drop[ Cases[DeleteCases[
    Table[{a - z, z, IsotopeData[{z, a}, "BindingEnergy"]}, {z, 1,118}, {a, IsotopeData[#, "MassNumber"] & /@ IsotopeData[z]}],
{_,Missing["Unknown"]}] // Flatten[#, 1] &, {nn, _, _}], None, {1}];
 nlm = NonlinearModelFit[Eb, a + b*(x* x/(nn*1. + x))*((x^2 + 1)*(1 + (0.0025 - 0.0003*Log[nn])*(g*x)^2))^(-.5), {a, b, g}, x];
 para = {para, -a /. nlm["BestFitParameters"]} // Flatten;
 RSq = {RSq, nlm["RSquared"]} // Flatten;
 MaxEr = {MaxEr, Last[Sort[nlm["MeanPredictionErrors"]]]} // Flatten;
 parc = {parc, b /. nlm["BestFitParameters"]} // Flatten;
 par = {par, g /. nlm["BestFitParameters"]} // Flatten, {nn, 2, 102}]
ListPlot[par, Filling -> Axis, AxesLabel -> {N, "g"},
  ImageSize -> {200, 200}] ListPlot[para, Filling -> Axis,
  AxesLabel -> {N, "a"}, ImageSize -> {200, 200}] ListPlot[parc,
  Filling -> Axis, AxesLabel -> {N, "b"}, ImageSize -> {200, 200}]
ListPlot[RSq, Filling -> Axis, AxesLabel -> {N, "RSquared"},
  ImageSize -> {200, 200}, DataRange -> Automatic] ListPlot[MaxEr,
  Filling -> Axis, AxesLabel -> {N, "MaxError"},
  ImageSize -> {200, 200}, DataRange -> Automatic]
```