

**СТРУКТУРА НЕЙТРОНА В ТЕОРИИ
КАЛУЦЫ-КЛЕЙНА**

**NEUTRON STRUCTURE IN KALUZA-KLEIN
THEORY**

Трунев Александр Петрович
к.ф.-м.н., Ph.D.
*Директор, A&E Trounev IT Consulting, Торонто,
Канада*

Alexander Trunev
Cand.Phys.-Math.Sci., Ph.D.
*Director, A&E Trounev IT Consulting, Toronto,
Canada*

На основе теории Калуцы-Клейна изучены особые состояния атома водорода, возникающие при взаимодействии протона со скалярным безмассовым полем. Показано, что некоторые состояния таких атомов имеют параметры нейтрона.

The special states of a hydrogen atom, arising from the interaction of a proton with a scalar massless field studied on the basis of Kaluza-Klein theory. It is shown that some states of the atoms have parameters of the neutron.

Ключевые слова: Теория Калуцы-Клейна, атом водорода, нейтрон, протон, электрон.

Keywords: KALUZA-KLEIN THEORY, Hydrogen atom, Electron, Proton, Neutron.

Introduction

The search for a metric that describes the elementary particles has been going on for over 100 years [1-6]. Pauli and Einstein [1] shown for both – 4-dimension and 5-dimension Kaluza metric [2], that there are no regular stationary solutions of the relativistic field equations. It has been found [7] that the effect of electromagnetic field on the metric in the five-dimensional space leads to a change in the mass spectrum of elementary particles. In particular, the interaction of protons with a scalar massless field can be formed of a particle with a mass close to the mass of the neutron. In this paper, we solve the problem of the structure of the neutron based on the models [7-8].

Description of the model

Following [7], we assume that a metric tensor in 5-dimensional space near the gravity massive center is represented as a power series of distance from the source $r = \sqrt{x^2 + y^2 + z^2}$, thus

$$G_{ik} = G_{ik}(0) + \dot{G}_{ik}(0)kr + \ddot{G}_{ik}(0)\frac{(kr)^2}{2} + \dots \quad (1)$$

Here the scale parameter k is given by the application of the model (1), and the dot denotes differentiation with respect to the dimensionless parameter $\tilde{r} = kr$. Consider the form of the tensor (1) resulting in retention of the first three terms in the expansion of the metric in the case of central force field with the gravitational potential in Newton's form.

Suppose $x^1 = ct, x^2 = x, x^3 = y, x^4 = z$, in this notation we have for the square of the interval in the 4-dimensional space:

$$ds^2 = (1 + 2j/c^2)c^2t^2 - (1 - 2j/c^2)(dx^2 + dy^2 + dz^2) \quad (2)$$

$$j = -\frac{gM}{r}$$

Here g - the gravitational constant, M - mass of the central body, c - speed of light. Assume that the coefficients of the metric in the 5-dimensional space are characterized by some parameter $e^2 = G_{11}(0) = -\mathcal{G}_{11}(0)$. Then, assuming that $e^2/k = 2gM/c^2$, we arrive at the expression of the interval depending on the parameters of the metric in the five-dimensional space:

$$ds^2 = (1 - e^2/kr)c^2t^2 - (1 + e^2/kr)(dx^2 + dy^2 + dz^2) \quad (3)$$

Further, note that in this case, the metric tensor in four dimensions is diagonal with components

$$g_{11} = 1 - e^2 / kr; \quad g_{22} = g_{33} = g_{44} = -(1 + e^2 / kr) \quad (4)$$

We define the vector potential of the source associated with the center of gravity in the form of

$$g_1 = e / kr, \quad \mathbf{g} = g_1 \mathbf{u} \quad (5)$$

Here \mathbf{u} is a three dimensional vector, which we define below. Hence, we find the scalar and vector potential of electromagnetic field

$$j_e = \frac{q}{r} = \frac{mc^2}{e} \frac{e}{kr}, \quad \mathbf{A} = j_e \mathbf{u} \quad (6)$$

Assuming $N = (kr)^2$ and calculating the metric tensor in 5-dimensional space, using (4) - (5), we find that in this case the expression (1) contains the right side only three members of the powers series

$$G_{ik} = \begin{pmatrix} Ng_{ik} + Ng_i g_k & Ng_k \\ Ng_i & N \end{pmatrix} = G_{ik}(0) + \dot{G}_{ik}(0)(kr) + \ddot{G}_{ik}(0) \frac{(kr)^2}{2} \quad (7)$$

Note that the zero term of (7), which describes a flat space, depends on the charge. In the metric (7), every massive body can have a positive or negative electric charge $q = \pm mc \sqrt{2gM / k} / e$. Since the charge is quantized, one can determine the mass, which generates an electron or a proton from the relations: $q = e$, $M = m$, therefore $m^3 = ke^4 / 2c^2 \gamma$. Hence we find the expression for the unknown parameter of the theory

$$k = 2gm^3 c^2 / e^4 \quad (8)$$

Note that this expression corresponds to the Coulomb law in the form (6) in the Gaussian system of units. In the SI system, the right-hand side of (8) should be

multiplied by $(4\pi\epsilon_0)^2$, where ϵ_0 - permittivity of vacuum. The numerical value of (8) having a dimension of inverse length, is in the case of an electron around $1.7 \cdot 10^{-28} m^{-1}$, and in the case of the proton about $1.05 \cdot 10^{-18} m^{-1}$. Note that the corresponding scale in the case of an electron exceeds the size of the observable universe, while for protons this scale is about 100 light-years.

This result shows that to describe the motion in the four-dimensional worlds, taking into account the forces of gravity and electromagnetism requires only a finite number of terms of (1) in the expansion of the metric tensor in 5-dimensional space.

Let us consider the lower limit of applicability of the developed model. For this we compare the zero and second terms in the expansion (7). Assuming that these terms have the same order, we find the corresponding minimum radius, which in the case of the electron coincides with its classical radius - Table 1. On this scale, the electrostatic field affects the 5-dimensional metric space, as shown below. Note that the minimum size of the proton coincides with the range of the weak interaction.

Table 1: Parameters of the metric tensor G_{ik} .

	$k, 1/m$	e	r_{max}, m	r_{min}, m
e-	1.703163E-28	4.799488E-43	5.87E+27	2.81799E-15
p+	1.054395E-18	1.618178E-36	9.48E+17	1.5347E-18

To calculate the contravariant tensor, we use the general expression of the form [7]

$$G^{ik} = N^{-1} \begin{pmatrix} g^{ik} & -g^k \\ -g^i & 1 + g^{ik} g_i g_k \end{pmatrix} \quad (9)$$

Calculating the contravariant components of the metric tensor and the four-vector potential, according to (5), we find that

$$\begin{aligned} g^{11} = a = (1 - e^2 / kr)^{-1}; \quad g^{22} = g^{33} = g^{44} = b = -(1 + e^2 / kr)^{-1} \\ g^1 = ag_1, \quad g^2 = bg_2, \quad g^3 = bg_3, \quad g^4 = bg_4 \end{aligned} \quad (10)$$

Hence, calculating the contravariant tensor according to equation (9), we obtain

$$G^{ik} = N^{-1} \begin{pmatrix} a & 0 & 0 & 0 & -g^1 \\ 0 & b & 0 & 0 & -g^2 \\ 0 & 0 & b & 0 & -g^3 \\ 0 & 0 & 0 & b & -g^4 \\ -g^1 & -g^2 & -g^3 & -g^4 & I \end{pmatrix} \quad (11)$$

It is indicated $I = 1 + ag_1^2 + b(g_2^2 + g_3^2 + g_4^2)$. Note that the contravariant components of the vector potential and the metric tensor in 4- and 5-dimensional space, proportional to the parameter a , have a singularity at the point, which corresponds to the gravitational radius $r = e^2 / k = 2gM / c^2$. The determinant of the metric tensor is equal to the inverse of the determinant of the contravariant tensor, which is easily calculated for the matrix (11); we have a result (see [7])

$$G = N^5 a^{-1} b^{-3} \quad (12)$$

To describe the motion of matter according to its wave properties, we assume that the standard Hamilton-Jacobi equation and relativistic mechanics, such as the Klein-Gordon equation in quantum mechanics arise as a consequence of the

wave equation in five-dimensional space [7]. This equation can in general be written as:

$$\frac{1}{\sqrt{-G}} \frac{\partial}{\partial x^m} \left(\sqrt{-G} G^{mm} \frac{\partial}{\partial x^n} \Psi \right) = 0 \quad (13)$$

Here - the wave function describing, according to (13), the massless scalar field in five-dimensional space.

Equation (13) is interesting because of it, by simple generalization we can derive all the basic models of quantum mechanics, including the Dirac equation, as in the nonrelativistic case, this equation reduces to the Schrödinger equation. From this we can also derive the eikonal equation, which is a 4-dimensional space is reduced to the Hamilton-Jacobi equation, which describes the motion of relativistic charged particles in electromagnetic and gravitational field [7].

Further, note that in the studied metric which depends only on the radial coordinate, the following relation

$$F^m = N \frac{\partial}{\partial x^m} (\sqrt{-G} G^{mm}) = N \frac{\partial r}{\partial x^m} \frac{d}{dr} (\sqrt{-G} G^{mm}) \quad (14)$$

In view of (10) (14), we write the wave equation (13) as

$$\frac{a}{c^2} \frac{\partial^2 \Psi}{\partial t^2} - |b| \nabla^2 \Psi + I \frac{\partial^2 \Psi}{\partial r^2} - 2g^i \frac{\partial^2 \Psi}{\partial x^i \partial r} + F^m \frac{\partial \Psi}{\partial x^m} = 0 \quad (15)$$

Note that the last term in equation (15) is of order $N^2 k = k^5 r^4 \ll 1$. Consequently, this term can be dropped in the problems, the characteristic scale which is considerably less than the maximum scale in Table 1. Equation (15) is remarkable in that it does not contain any parameters characterizing the scalar field. Field acquires a mass and charge (not only electrical but also the strong and

weak [7]) in the interaction with the central body, which is due only to the metric and the 5-dimensional space.

The spectrum of atomic particles with axial symmetry

Consider the motion of matter around a charged center of gravity, which has an electrical charge and strong charge, for example, around the proton. In the process of solving this problem it is necessary to define the inertial mass of matter and energy ties. Since equation (15) is linear and homogeneous, this problem can be solved in general.

We introduce a polar coordinate system (r, f, z) with the z axis is directed along the vector potential (6), we put in equation (15)

$$\Psi = y(r) \exp(ilf + ik_z z - i\omega t - ik_r r) \quad (16)$$

Separating the variables, we find that the radial distribution of matter is described by the following equation (here rejected, because of its smallness, the last term in equation (15)):

$$-\frac{aw^2}{c^2}y - b \left(y_{rr} + \frac{1}{r}y_r - \frac{l^2}{r^2}y - k_z^2 y \right) - \lambda k_r^2 y + 2g^1 c^{-1} \omega k_r y - 2g^z k_z k_r y = 0 \quad (17)$$

We assume that the characteristic scale of the spatial distribution of matter far beyond the gravitational radius $r \gg \varepsilon^2/k = 2\gamma M/c^2$. Then in the first approximation we can assume that $a \approx -b \approx 1$; $I = 1 + g_1^2 - g^2 \approx 1$. We also use the definition of the vector and scalar potential (5), as a result we obtain

$$y_{rr} + \frac{1}{r}y_r - \frac{l^2}{r^2}y - k_z^2y + \left(K^2 + \frac{k_g}{r}\right)y = 0 \quad (18)$$

$$K^2 = k_r^2 + w^2/c^2, \quad k_g = -2ek_r(k_z u_z + w/c)/k > 0$$

Note that equation (18) coincides in form with what was obtained in [8-9] in the case of axially symmetric solutions of the Schrodinger equation describing the special states of the hydrogen atom. We seek the solution of equation (18) as

$$y = y_0 \frac{\exp(-r/r_0)}{r^a} \quad (19)$$

Substituting (19) into equation (18), we have

$$\frac{a^2 - l^2}{r^2} + \frac{2a - 1 + r_0 k_g}{rr_0} + \frac{1}{r_0^2} - k_z^2 + k_r^2 + \frac{w^2}{c^2} = 0 \quad (20)$$

Equating coefficients of like powers of r, we obtain the equations for determining the unknown parameters:

$$a = \pm l, \quad r_0 = \frac{1 - 2a}{k_g}, \quad \frac{1}{r_0^2} - k_z^2 + k_r^2 + \frac{w^2}{c^2} = 0 \quad (21)$$

The second equation (21) holds only for values of the exponent, for which the inequality $a < 1/2$ is true. Hence, we find an equation for determining the frequency as

$$\frac{4e^2 k_r^2}{k^2 (2l + 1)^2} \left(k_z u_z + \frac{w}{c}\right)^2 - k_z^2 + k_r^2 + \frac{w^2}{c^2} = 0 \quad (22)$$

It should be noted that the original metric in the five-dimensional space defined by metric tensor (7), which depends only on the parameters of the central body, i.e. the charge and mass of the proton, so the left-hand side of equation (22) should be placed $e/k = e^2/m_p c^2$.

State with axial symmetry in the Schrödinger quantum mechanics

Consider the problem of axially symmetric states of the hydrogen atom in quantum mechanics, Schrödinger [8]. The equation describing the stationary states of an electron with energy E in an external potential field $U = -\mathbf{ahc}/r$ (here put the fine structure constant $\mathbf{a} = 1/137,035999679$) in a cylindrical coordinate system is as follows:

$$\Psi_{rr} + \frac{1}{r}\Psi_r + \frac{1}{r^2}\Psi_{jj} + \Psi_{zz} + \frac{2m}{\mathbf{h}^2}(E - U)\Psi = 0 \quad (23)$$

We seek the solution of equation (23) as

$$\Psi = \mathbf{y}(r) \exp(ilj + ik_z z) \quad (24)$$

Substituting (24) into equation (23), we obtain

$$\mathbf{y}_{rr} + \frac{1}{r}\mathbf{y}_r - \frac{l^2}{r^2}\mathbf{y} - k_z^2\mathbf{y} + \frac{2m}{\mathbf{h}^2}\left(E + \frac{\mathbf{ahc}}{r}\right)\mathbf{y} = 0 \quad (25)$$

Comparing equations (25) and (18), we find their full identity, so the solution of equation (25) can also be found in the form (19). In this case the system of equations for determining the parameters similar to (21)

$$a = \pm l, \quad r_0 = \frac{1 - 2a}{2a} \frac{\mathbf{h}}{mc}, \quad \frac{1}{r_0^2} - k_z^2 + \frac{2mE}{\mathbf{h}^2} = 0 \quad (26)$$

The equation for the energy in this case has the form

$$\frac{4a^2}{(1 + 2l)^2} \frac{m^2 c^2}{\mathbf{h}^2} - k_z^2 + \frac{2mE}{\mathbf{h}^2} = 0 \quad (27)$$

It is believed that the energy states of a hydrogen atom depend on the reduced mass $m = m_e m_p / (m_p + m_e) \approx m_e$. In the following analysis we use this mass scale.

Parameters of the 5-dimensional atom

Bound states of the hydrogen atom have the energy, which is represented by the first term in equation (22) and (27). Since we are considering the same atom, we equate these terms, pre-multiplying them by a constant factor $\mathbf{h}^2 / 2m_1$. As a mass scale will be used in equation (27) the reduced mass $m = m_e m_p / (m_p + m_e) \approx m_e$, and in equation (22) a mass scale m_1 , which we define below, we have

$$\frac{\mathbf{h}^2 e^2}{m_1 k^2} k_r^2 (k_z u_z + w/c)^2 = a^2 m c^2 \quad (28)$$

Note that a similar equation was obtained in [9], although in this paper we give an exact solution of the original equations, and in [9] - approximate solution based on the assumption of the existence of classical trajectories.

Using the parameters of the metric, we reduce equation (28) to the form

$$(k_z u_z + w/c)^2 = \frac{m_p^2 c^2}{\mathbf{h}^2} \frac{m_1 m c^2}{\mathbf{h}^2 k_r^2} \quad (29)$$

Since the wave vector k_z is arbitrary, and the frequency must satisfy the equation (22) and equation (29), we must assume that this is achieved by selecting k_r . Expressing this parameter from equation (29) and substituting it into equation (22), we find from (28)

$$\frac{4a^2 m_1 m}{\mathbf{h}^2 (2l+1)^2} + \frac{m_p^2 m_1 m}{\mathbf{h}^4} \left(k_z u_z + \frac{w}{c} \right)^{-2} - k_z^2 + \frac{w^2}{c^2} = 0 \quad (30)$$

The first term in equation (30) describes the electromagnetic interaction is small compared with the others, so it can be neglected, resulting in equation (30) reduces to

$$\frac{m_p^2 c^2}{\mathbf{h}^2} \frac{m_1 m c^2}{\mathbf{h}^2} = \left(k_z^2 - \frac{w^2}{c^2} \right) \left(k_z u_z + \frac{w}{c} \right)^2 \quad (31)$$

The simplest result is obtained in the absence of magnetic interaction, i.e. at $u_z = 0$. In this case, we find from equation (31)

$$\frac{w}{c} = \pm \sqrt{\frac{k_z^2}{2}} \pm \sqrt{\frac{k_z^4}{4} - \frac{m_p^2 m m_1 c^4}{\mathbf{h}^4}} \quad (32)$$

It follows that the scalar field has a frequency boundary and the boundary values of the wave vector, which is determined from (32) according to

$$W_e = W|_{k_z=k_e} = \pm \frac{c^2 \sqrt{m_p \sqrt{mm_1}}}{\mathbf{h}}, \quad k_e = \pm \frac{c \sqrt{2m_p \sqrt{mm_1}}}{\mathbf{h}} \quad (33)$$

Finally, assuming that the lower cutoff frequency corresponds to the effective mass of an electron $\mathbf{h}w = \pm mc^2$, we find the unknown mass scale

$$m_1 = m^3 / m_p^2 \approx 2,96285 \cdot 10^{-7} m_e \quad (34)$$

The rest energy of which $m_1 c^2 = 0,151319 eV$ corresponds to a temperature of about 1756K.

The wave vector of the fifth dimension is $k_r = \pm mc / \mathbf{h}$, the sign is determined in accordance with the third equation (18), in which the parameter of interaction put as $k_g > 0$. The distribution of density in the toroid atom, calculated from equation (19) for $l = 1$ is shown in Fig. 1.

In Fig. 2 the relationship between momentum and energy in Lorentz's theory, in five-atom, and for electromagnetic radiation are shown. Note that the dispersion relation for the five-atom has four branches. In Fig. 2 shown only two branches, which correspond to positive values of energy - (32) with a positive sign before the radical, and with the signs + and - under the radical, respectively. Note that for the excitation of vibrations in five-dimensional atom need to make the minimal momentum $p = \sqrt{2}mc$, which corresponds to the energy $E = mc^2$.

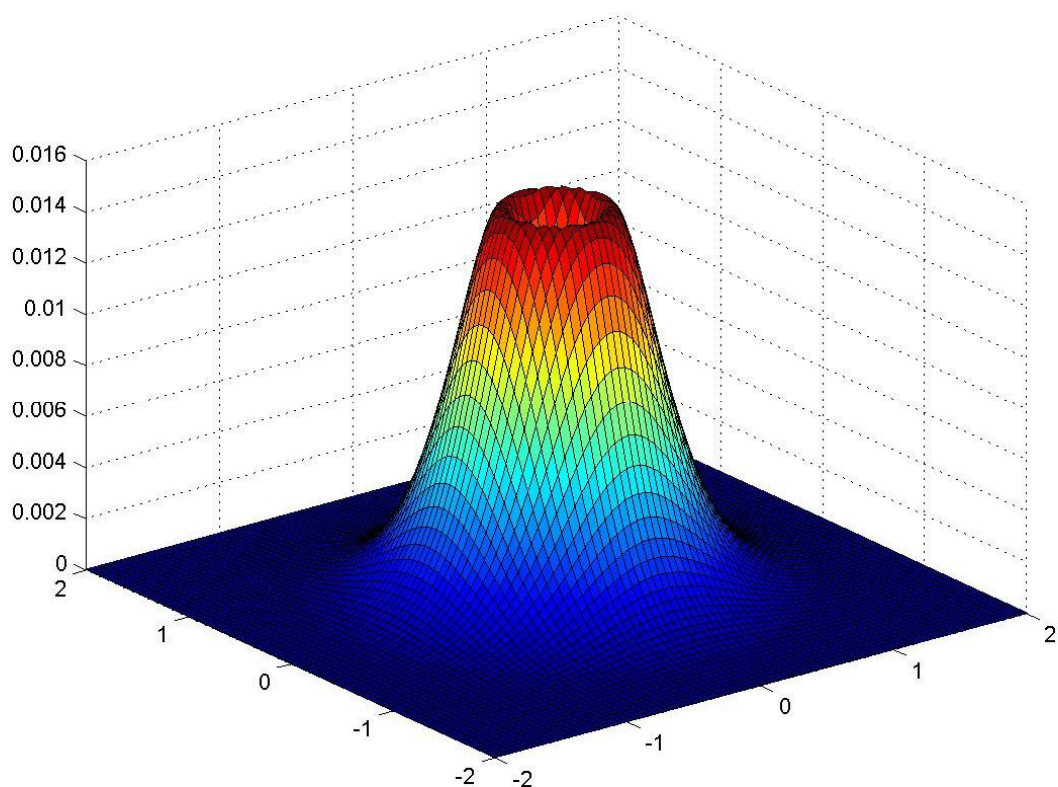


Figure 1: The density distribution in a toroid atom, calculated from equation (19) at $l = 1$.

Interestingly, the dispersion relation in the five-atom, along with the ascending branch $E = cp$, has a descending branch, for which higher values momentum $p \gg \sqrt{2}mc$ correspond to low energy $E \ll mc^2$. If such a branch of the dispersion relation does exist, then it opens up great possibilities in the jet engines new generation.

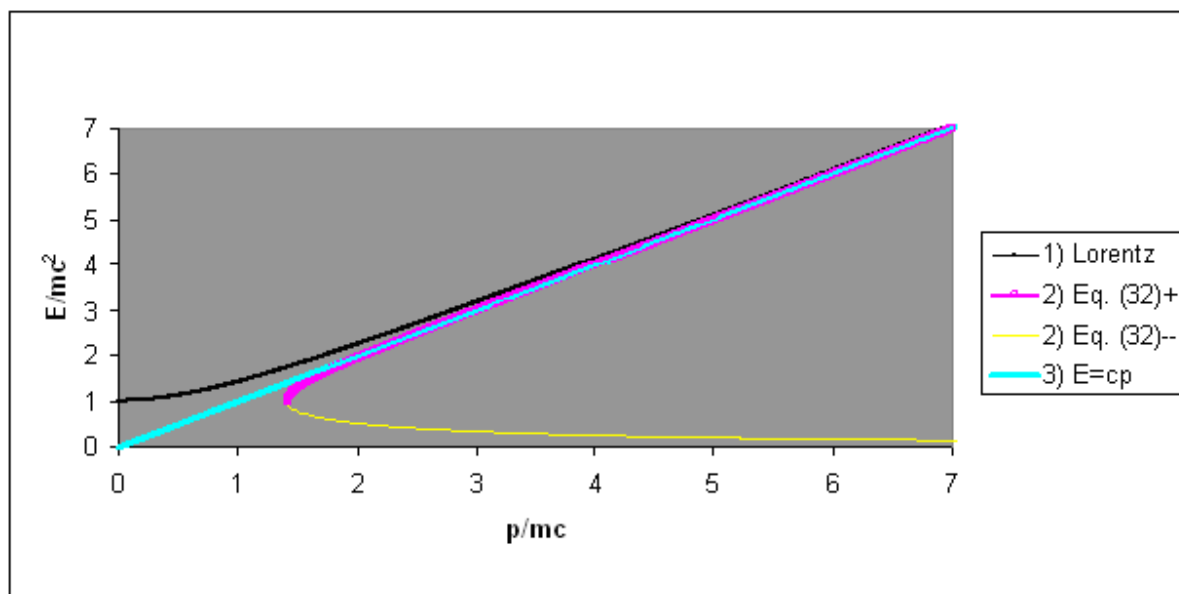


Figure 2: Energy-momentum dispersion relation: 1) Lorentz theory; 2) the five-atom Eq. (32); 3) electromagnetic radiation $E = cp$.

Thus, the set of electron states in a hydrogen atom has analogy with an axial symmetry in 5-dimensional atom, consisting of a proton and a scalar massless field. Based on this analogy, the parameters of five-atom dispersion relations (30) - (32) have been derived. Note that the dispersion relation (32) does not contain the interaction parameters, so it is applicable not only for atoms but also for the free particles - electrons in a particular state.

The structure of the neutron

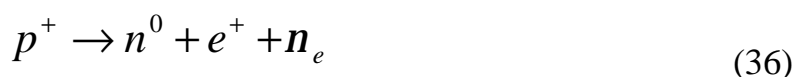
The experimentally determined main properties of the neutron are shown in Table 2. The average lifetime of a free neutron is about to 885.7 c. A neutron decays into a proton, electron and antineutrino in the scheme (beta decay):



Table 2: The fundamental properties of elementary particles involved in the reaction (35) [10].

Elementary particle	Neutron n^0	Proton p^+	Electron e^-	\bar{n}_e
Mass, MeV/c ²	939.565560(81)	938.272013(23)	0.510998910(13)	< 2.2 eV
Charge, Coulomb	0	$1.602176487(40) \times 10^{-19}$	$-1.602176487(40) \times 10^{-19}$	0
The magnetic moment, nuclear magneton (N) or Bohr magneton (B)	$-1.9130427(5) \mu\text{N}$	$2.792847351(28) \mu\text{N}$	$-1.00115965218111 \mu\text{B}$	$10^{-19} \mu\text{B}$
Electric dipole moment	$<2.9 \times 10^{-26} \text{ e.cm}$	$<5.4 \times 10^{-24} \text{ e.cm}$?	?

It was found that the proton in the nucleus may be transferred in accordance with the scheme of the neutron inverse beta-decay



Another possible channel is the electron capture:



The first theory of beta decay proposed in 1933 by Enrico Fermi. Later it was suggested several theories, including the theory of Feynman and Gell-Mann [11]. At the present time, according to the present standard model, the reaction (35) comes with the participation of the intermediate vector gauge boson W^- [12]. In

this model, protons and neutrons are composite particles containing three quarks on. However, protons are split into their component parts have failed, although it is believed that hadronic jets observed in experiments on collisions of protons at high energy, are the quark-gluon plasma [13].

It was found that the distribution of electric charge in the neutron consists of a negatively charged outer coat, inner layer of positively charged and negatively charged nucleus [14]. From the decay scheme (35) and classical representations of the interaction of charged particles, one would assume that the proton forms together with the electron kind of hydrogen atom, which explains the observed electromagnetic structure of the neutron [15]. But we know that the state describing the hydrogen atom with a large binding energy, consistent hydrino [16-18]. In these states, the mass of the hydrogen atom is different from that of the proton by a small amount $am_e c^2$ that is not consistent with the large mass of the neutron, exceeding the total mass of a proton and an electron by an amount $(m_n - m_p - m_e) / m_e = 1,531015$.

Consider the five-state hydrogen atom, which correspond to the parameters of the neutron and proton in Table 1. In this case, there are no similar solutions, which are described a neutron on the basis of relativistic Dirac or Klein-Gordon equation. The wave vector of the fifth dimension can be determined from the third equation (22), as a result we find

$$S = \frac{P^2 - E^2}{1 + b(Pu + E)^2}, k_r = \pm \frac{m_e c}{\hbar} \sqrt{S}$$

$$b = \frac{4e^2}{\hbar^2 k^2} \frac{m_e^2 c^2}{(1 - 2a)^2}, P = \frac{\hbar k_z}{m_e c}, E = \frac{\hbar \omega}{m_e c^2} \quad (38)$$

Surface, which is given by the first equation (38), depends on the interaction parameter, which in turn depends on the type of interaction. In general, we can set, but the square of the charge can take three values of [7], which correspond to the electromagnetic, strong and weak interactions, respectively - see Table 3.

Table 3: The parameter b for the three types of interaction with a = 0

Interaction type	Charge	Parameter of Interaction, b
Electromagnetic	$e^2 = \mathbf{a}\hbar c$	6.3179E-11
Strong	$e_s^2 = e^2 (m_p / m_e)^{3/2}$	4.97091E-06
Weak	$e_w^2 = e^2 (m_e / m_p)^{3/2}$	8.0299E-16

As follows from Table 2, the effect of the interaction parameter on the dispersion relation, even in the case of strong interaction manifests itself at energies of about 300 electron masses. There is however a special case where $a \rightarrow 1/2$. Then, it follows from (38) the interaction parameter can take any value. In this particular case, all interactions are compared with each other in the sense that there always exists a value of the exponent, for any type of interaction we have the product parameters.

The surface $S = S(P, E)$ for parameter values $b = 0.039026$; $u = u_z = 1$ is shown in Figure 3. Each section of the surface for positive values of S determines the line of dispersion equation: $E = E(P, S)$.

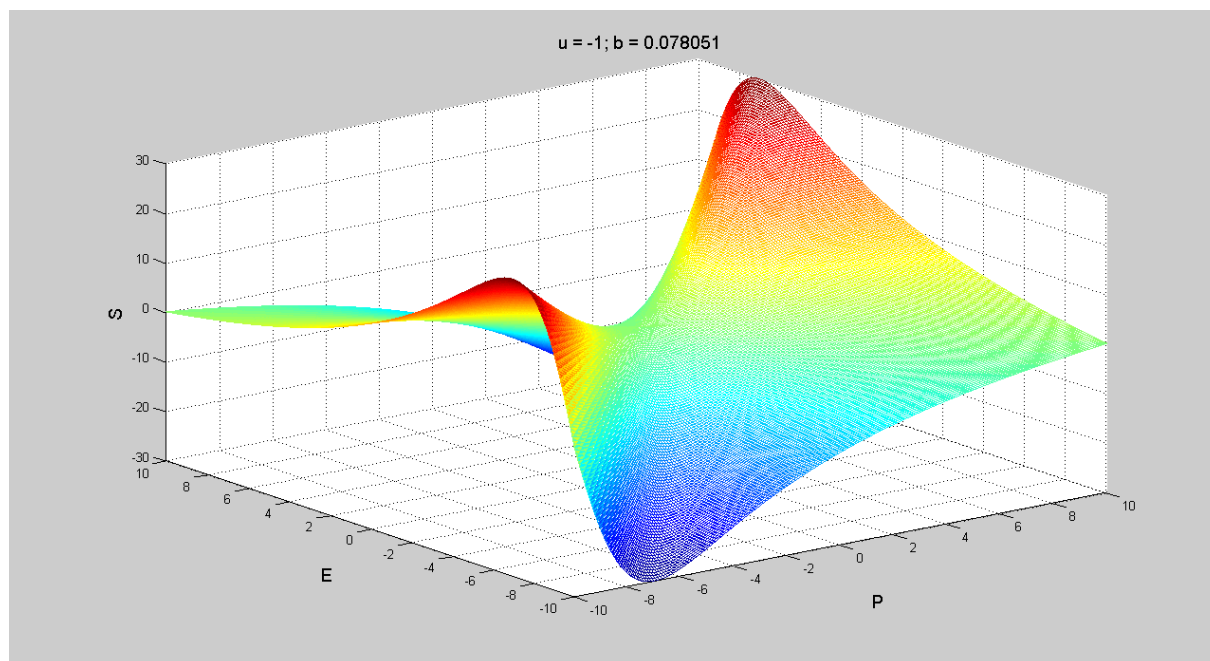


Figure 3: Surface, which characterizes the special states of a hydrogen atom at $b = 0,078051$; $u = u_z = -1$.

Note that a special case $a = 1/2$ was considered for the Schrödinger equation in [8] and equation (18) in [9]. The common property of these states lies in the fact that the electron approaches the nucleus for a short distance of the order of the classical electron radius. For example, in the model [15] we have

$$r_0 / r_e = 0.4777778, \quad r_e = e^2 / m_e c^2, \quad L = 1.376791 \mathbf{a h} \quad (39)$$

With this approximation may form a neutron? Consider the dispersion relation that characterizes this condition. Solving the first equation (38) with respect to energy, we find the dispersion relation - Fig. 4, which allows

determining the minimum energy and momentum of a scalar field in a particular state, using the conditions for the ascending part of the spectrum:

$$E_m = (m_n - m_p) / m_e \approx 2.531015; \quad \lim_{P \rightarrow \infty} E/P = 1$$

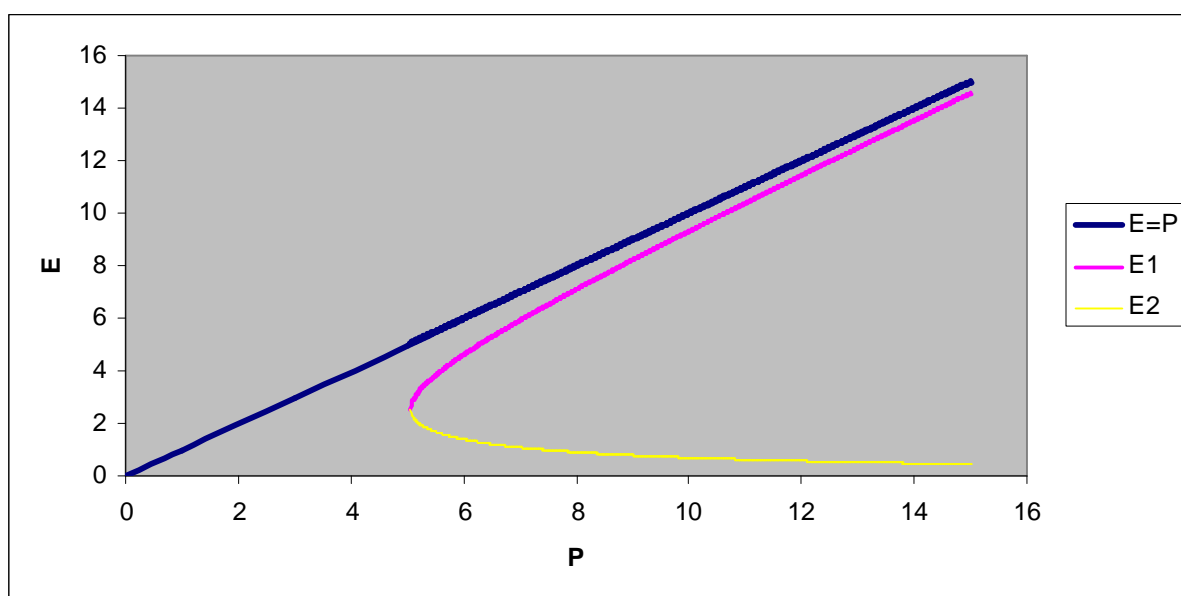


Figure 4: Energy-momentum dispersion relation which characterizes the special states of a hydrogen atom at $b = 0,078051$; $u = u_z = -1$.

These conditions allow us to determine the numerical values of other parameters

$$E_m = 2.531015; \quad P_m = 5.06203; \quad S = 12.81208; \quad b = 0.078051 \quad (40)$$

Note that the dispersion curves in Fig. 2 and 4, describing the special states of the hydrogen atom are similar, since both contain an ascending and descending branch, as well as maximum limit of the spectrum. For the excitation of these states must report a minimum momentum, i.e. their occurrence is possible, for

example, in the collision of solid bodies. The difference between them is the same in that the states described by equations (28-32), the wave vector in the fifth dimension is defined in the process of solving the problem, whereas the parameters of the state of the dispersion relation shown in Fig. 4, are determined from the equation (48), in which the wave vector in the fifth dimension is given constant. The size of the hydrogen atom in this state is determined by the Compton wavelength of an electron:

$$r_0 / l_e = 1 / E \approx 0,395098, \quad l_e = \mathbf{h} / m_e c \quad (41)$$

As it known, the state of the hydrogen atom, have a characteristic size (41), associated with hydrino [7-8, 16-18]. For the first time, these states were obtained by Sommerfeld in 1923 as a solution of the Klein-Gordon equation for the relativistic hydrogen atom. Note that the Sommerfeld solution can be obtained on the basis of equation (17), provided, that in the absence of magnetic interaction. At present there is not just a theory, but a lot of experiments confirming the hypothesis of the existence of special states of the hydrogen atom - hydrino [18]. Solution obtained above is a generalization of known results [16-17] to the case of magnetic interaction caused by the Kerr metric [4].

Note that the dispersion relation $S = S(P, E)$ obtained by section of the surface shown in Fig. 3, is invariant with respect to the choice of scale. Therefore, choosing a scale of the classical electron radius, we obtain

$$r_0 / r_e = 1 / E \approx 0,395098, \quad r_e = e^2 / m_e c^2 \quad (42)$$

This result is consistent with the data (39), but the final choice of scale in the model depends on the determination of the neutron magnetic moment [15], which is beyond the scope of this paper.

Thus, we have shown that there are specific states of the hydrogen atom, which describe a particle with mass and size of the neutron. These states arise in the interaction of protons with a massless scalar field. The resulting interaction density distribution of the scalar field corresponds to the Yukawa potential

$$y^2 = y_0^2 \frac{\exp(-2r/r_0)}{r^{1-d}}, \quad d = 1 - 2a \ll 1$$

Further study of this problem may be related with the influence of the scalar field on a standard metric in Kaluza-Klein theory [5].

References

1. Einstein A., Pauli W.— Ann of Phys., 1943, v. 44, p. 131. (см. Альберт Эйнштейн. Собрание научных трудов. Т. 2. – М., Наука, 1966, статья 123).
2. Kaluza, Theodor. Zum Unitätsproblem in der Physik. Sitzungsber. Preuss. Akad. Wiss. Berlin. (Math. Phys.) **1921**: 966–972.
3. K. Lake. Reissner-Nordstrom-de Sitter metric, the third law, and cosmic censorship // Phys. Rev. D 19, 421 (1979).
4. Alexander Burinskii. Kerr Geometry as Space-Time Structure of the Dirac Electron // arXiv:0712.0577v1 [hep-th], 4 Dec 2007
5. [V. Dzhunushaliev](http://arxiv.org/abs/gr-qc/0405017v1). Wormhole solutions in 5D Kaluza-Klein theory as string-like objects // arXiv:gr-qc/0405017v1
6. Jorma Louko, Robert B. Mann, Donald Marolf. Geons with spin and charge // <http://arxiv.org/abs/gr-qc/0412012v2>
7. Трунев А.П. Фундаментальные взаимодействия в теории Калуцы-Клейна // Научный журнал КубГАУ. – Краснодар: КубГАУ, 2011. – №07(71). С. 502 – 527. – Режим доступа: <http://ej.kubagro.ru/2011/07/pdf/39.pdf>
8. Alexander Trunev. Electron structure, hydrino and cold fusion // Chaos and Correlation, № 11, Nov. 25, 2011, http://chaosandcorrelation.org/Chaos/CR11_2011.pdf
9. Alexander Trunev. Electron structure in Kaluza-Klein theory // Chaos and Correlation, №12, Dec. 7, 2011 http://chaosandcorrelation.org/Chaos/CR12_2011.pdf
10. Mohr P.J., Taylor B.N., Newell D.B. CODATA recommended values of the fundamental physical constants // Reviews of Modern Physics 80: 633–730. 2006.
11. Richard P. Feynman. The Theory of Fundamental Processes. Addison Wesley. ISBN 0-8053-2507-7. (1961).
12. J. Christman. The Weak Interaction. Physnet. Michigan State University, 2001. http://physnet2.pa.msu.edu/home/modules/pdf_modules/m281.pdf
13. Hunting the Quark Gluon Plasma. RESULTS FROM THE FIRST 3 YEARS AT RHIC. ASSESSMENTS BY THE EXPERIMENTAL COLLABORATIONS. Relativistic Heavy Ion Collider (RHIC). BNL -73847-2005, April 18, 2005.

14. John Arrington, Kees de Jager and Charles F. Perdrisat. Nucleon Form Factors-A Jefferson Lab Perspective// http://arxiv.org/PS_cache/arxiv/pdf/1102/1102.2463v1.pdf
15. Alexander Trunev. Neutron decay in the classic and quantum mechanics//Chaos and Correlation, April 30, 2011, http://chaosandcorrelation.org/Chaos/CR_4_2011.pdf
16. Naudts, Jan (5 August 2005). On the hydrino state of the relativistic hydrogen atom. arXiv:physics/0507193.
17. Dombey, Norman (8 August 2006). "The hydrino and other unlikely states". Physics Letters A 360: 62. arXiv:physics/0608095
18. Mills, Randell L. The Grand Unified Theory of Classical Physics. Blacklight Power. <http://www.blacklightpower.com/theory/bookdownload.shtml>. Retrieved 2009-08-15.