

УДК 532.526.4

UDC 532.526.4

**ТЕОРИЯ И КОНСТАНТЫ ПРИСТЕННОЙ
ТУРБУЛЕНТНОСТИ****THEORY AND CONSTANTS OF WALL
TURBULENCE**

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В работе представлена полностью замкнутая модель турбулентного пограничного слоя, полученная из уравнения Навье-Стокса. Фундаментальные константы пристенной турбулентности, включая постоянную Кармана, определены из теории. Эта модель была развита для ускоренного и неизотермического пограничного слоя. Профили средней скорости и температуры, вычисленные на основе модели находятся в согласии с экспериментальными данными

The completely closed model of wall turbulence was derived directly from the Navier-Stokes equation. The fundamental constants of wall turbulence including the Karman constant have been calculated within a theory. This model has been developed also for the accelerated and non-isothermal turbulent boundary layer flows. The estimated mean velocity, temperature and impurity concentration profiles as well as the spectral characteristics of the streamwise velocity component are to be shown in a good agreement with the experimental data

Ключевые слова: ТУРБУЛЕНТНЫЙ
ПОГРАНИЧНЫЙ СЛОЙ, ЛОГАРИФМИЧЕСКИЙ
ПРОФИЛЬ

Keywords: TURBULENT BOUNDARY LAYER,
LOGARITHMIC PROFILE

1. Introduction

Nikuradse [1] found that the turbulent boundary layer structure includes the region where the mean velocity increases with the distance from the wall as a logarithmic function

$$\frac{U}{u_t} = \frac{1}{k} \ln \frac{u_t z}{n} + c_0 \quad (1)$$

where u_t is the friction velocity, $u_t = \sqrt{t/r}$, t is the wall shear stress, r is the fluid density, z is the distance from the wall, n is the kinematic viscosity. Nikuradse [1] established that $k = 0.4$, $c_0 = 5.5$ for turbulent flows in smooth pipes.

The logarithmic law (1) has been confirmed by many and many researchers of turbulence [2-7, 13-14,17]. Recently it was shown [5] that the log law can be excellent agreed with the experimental pip flow data for $600 < zu_t/n < 0.07Ru_t/n$, where R is the pip radius. For this range the log law constants were shown to be $k = 0.436$, $c_0 = 6.15$. Also the log law was confirmed for a zero pressure gradient turbulent boundary layer flow in the precise wind-tunnel experiment [6] and the constants were obtained $k = 0.38$, $c_0 = 4.1$. To estimate the Karman constant the normalised local slope of the mean velocity profile has been used as follows $\Xi = d \ln z^+ / dU^+$ where $z^+ = zu_t/n$ is the inner layer variable - see Figure 1a.

So this universal law can be considered as a fundamental fact of the theory of turbulence. Nevertheless it may be wonder that this law never been derived from "first principles" excepting the similarity theory (see the typical arguments in ref. [2-8] and other).

To derive this law we can begin with simple assumption that the logarithmic profile of velocity is a fragment in solution of hydrodynamics equations of viscous fluid in a form of Navier-Stokes. Thus the way a question is put can be different: how the initial equations should be transformed so that in their solution boundary layer should contain logarithmic profile? This type of transformation was developed recently [9-11]. The idea of transformation is rather simple and as a matter, was driven at bringing into initial system equations of some set of random parameters, describing fluctuations of conventional boundary of viscous sublayer.

To understand this idea we should note that the logarithmic distribution (1) is a typical mean velocity profile measured in the turbulent flow bounded by a smooth wall. In a case of the turbulent flow over a rough wall Nikuradse [12] established (for sand-roughened pipes) that if the roughness height significantly exceeds the viscous sublayer thickness, then the mean velocity profile can be described by the logarithmic function as follows:

$$\frac{U}{u_t} = \frac{1}{k} \ln \frac{z}{k_s} + c_s \quad (2)$$

where k_s is the characteristic scale of the sand roughness, k, c_s are empirical values (Nikuradse found that $k = 0.4, c_s = 8.5$ for the completely rough regime).

The sentence "the roughness height significantly exceeds the viscous sublayer thickness" means only that $k_s \gg n/u_t$ or in another form $k_s^+ = k_s u_t / n \gg 1$. Obviously that k_s^+ is the roughness Reynolds number and n/u_t is the characteristic thickness of viscous sublayer. Compared equations (1) and (2) one can conclude that there is an universal model described the turbulent boundary layer flows over smooth and rough surfaces. As it was shown [9-11] this model can be derived directly from the Navier-Stokes equation and that the logarithmic distribution is an asymptotic solution of this model.

2 Principles of theory

Let $r = r(x, y)$ is a function describing the roughness geometry and $\tilde{h} = \tilde{h}(x, y, t)$ is the thickness of viscous sublayer. The *dynamic roughness surface* can be defined as follows $h(x, y, t) = r(x, y) + \tilde{h}(x, y, t)$. Thus the mean dynamic roughness height is given by $\bar{h} = k_s + I_0^+ n / u_t$, where I_0^+ is the dimensionless thickness of viscous sublayer. It should be noted that in the case of a smooth plate for

$r(x, y) = 0$ the dynamic roughness surface equals to the viscous sublayer conventional surface i.e. $h(x, y, t) = \tilde{h}(x, y, t)$.

In Figure 1b the frequency diagram [13] of instantaneous thickness of a viscous sublayer in the turbulent boundary layer over a smooth surface is shown. The maximum of frequency diagram is established for $z^+ = z_0^+ \approx 9$. The first maximum of the inverse normalised local slope of the mean velocity profile $A = dU^+ / d \ln z^+$ also locates approximately at $z^+ \approx 9$ - see Figure 1a. Thus one can put $I_0^+ \approx 9$.

For the flow over a smooth surface we have $\bar{h} = I_0^+ n / u_t$ and in this case the logarithmic velocity distribution in the form (1) is realised. When $k_s \gg I_0^+ n / u_t$ and therefore $\bar{h} \approx k_s$ then the mean velocity profile can be described by eq. (2). This two cases can be described automatically if the flow velocity vector, $\mathbf{u} = (u, v, w)$, can be written as follows

$$\mathbf{u} = \mathbf{u}(x, y, z/h(x, y, t), t) \tag{3}$$

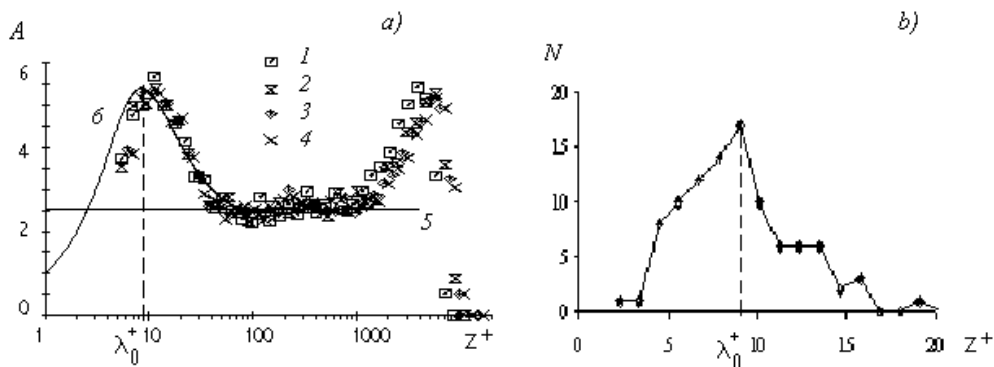


Figure 1: a) The inverse normalised local slope of the mean velocity profile $A = dU^+ / d \ln z^+$ estimated from the van Driest model [14]- the solid line 6 and from the experimental data by Österlund [6] (1-4). Horizontal solid line (5) corresponds to the estimated quantity $A = 2.54$; b) the frequency diagram of instantaneous thickness of viscous sublayer in the turbulent boundary layer over a smooth wall [13].

If the streamwise velocity has a logarithmic asymptotic, $u \propto \ln(z/h)$, then also the mean velocity has the logarithmic asymptotic, $U = \langle u \rangle \propto \ln(z/\bar{h})$.

Unfortunately this very simple idea can't be used directly by substituting expression (3) in the Navier-Stokes equation, because the dynamic roughness surface can't be described by a regular function in a common case.

Therefore the dynamic roughness surface is advised to characterise by the set of the random continuous parameters h, h_t, h_x, h_y , with a well-known function of distribution $f_s = f_s(h, h_x, h_y, h_t)$. The reason why only the first odder derivatives of the function $h = h(x, y, t)$ play role in this theory is explained by the volume averaging used for a filtration of random functions.

We suppose that $h = z / h = const$ and let us consider a representative region of flow by a volume $dV = L_x L_y dz$, where L_x, L_y are typical scales of flow in the x, y directions accordingly. Then let us consider subregion dV_s lying in the representative region of the flow, dV , in which the random parameters h, h_t, h_x, h_y are changed in the intervals $(h; h + dh)$, $(h_t; h_t + dh_t)$, $(h_x; h_x + dh_x)$, $(h_y; h_y + dh_y)$ accordingly. The volume of this subregion is given by

$$dV_s = dV f_s(h, h_x, h_y, h_t) dh dh_x dh_y dh_t.$$

The random function describing the flow velocity can be determined by the totting expression $\mathbf{u} = \mathbf{u}(x, y, z/h(x, y, t), t)$ in the volume dV_s :

$$\tilde{\mathbf{u}}(h, t, h, h_x, h_y, h_t) = \lim_{dV \rightarrow dV_s} \frac{1}{dV} \int_{dV} \mathbf{u}(x, y, h, t) dx dy dz$$

where dV is an arbitrary volume enclosed in $dV = L_x L_y dz$ and containing dV_s as a whole.

Obviously, $\tilde{\mathbf{u}}(h, t, h, h_x, h_y, h_t)$ is the random function, because it depends on the random parameters. The equations describing dynamics of random functions $\tilde{\mathbf{u}} = \tilde{\mathbf{u}}(h, t, h, h_x, h_y, h_t)$ immediately follow from the hydrodynamic equations of viscous fluid [9-11]. Statistical moment of an order m of the random function $\tilde{\mathbf{u}}(h, t, h, h_x, h_y, h_t)$ are determined as follows

$$\bar{\tilde{\mathbf{u}}}^m(z, t) = \int \tilde{\mathbf{u}}^m(h, t, h, h_x, h_y, h_t) f_s(h, h_x, h_y, h_t) dh dh_x dh_y dh_t.$$

This algorithm is a very effective in the wall turbulence problem because in this case the random functions $\tilde{\mathbf{u}}(h, t, h, h_x, h_y, h_t)$ with fixed variables h, h_x, h_y, h_t have the properties which are similar to the mean flow parameters. Otherwise for the solutions presented for instance by the logarithmic function we can suppose that

$$\bar{\tilde{\mathbf{u}}} = \int \tilde{\mathbf{u}}(h, t, h, h_x, h_y, h_t) f_s(h, h_x, h_y, h_t) dh dh_x dh_y dh_t = \tilde{\mathbf{u}}(z/h_*, h_x^*, h_y^*, h_t^*),$$

where the parameters with stars can be estimated from the comparison of solutions with experimental data or calculated from a theory considered below.

A zero pressure gradient turbulent boundary layer over a smooth surface can be described by the completely closed equation system derived directly from the

Navier-Stokes equation (see [10-11] for details). Utilised the inner layer variables this model can be written in the form

$$\frac{du^+}{dz^+} = e^{-I} \left(\frac{\cos^2 a}{1+x^2} + \frac{\sin^2 a}{\sqrt{1+x^2}} \right), \quad (4)$$

$$R_t c \frac{d^2 c}{dx^2} + \frac{d}{dx} (1+x^2) \frac{d^2 c}{dx^2} = 0, \quad I = R_t \int_0^x \frac{c dx}{1+x^2}$$

where $u^+ = \tilde{u}/u_t$, $x = z^+/I^+$, $I^+ = hu_t/n\sqrt{h_x^2+h_y^2}$ is the characteristic dimensionless scale of the viscous sublayer, $a = \arctan(h_y/h_x)$, $v^+ = \tilde{v}/u_t$, $w^+ = \tilde{w}/u_t$, $c = [(u^+ \cos a + v^+ \sin a + w_0^+)x - w^+]/w_0^+$, $w_0^+ = h_t/u_*\sqrt{h_x^2+h_y^2}$ is the second scale of velocity in the wall turbulence, $R_t = hh_t/n(h_x^2+h_y^2)$ is the Reynolds number calculated on the dynamical roughness parameters (it should be noticed that the random parameters R_t, w_0^+, a, I^+ are not functions of time or space variables!).

The boundary conditions for Eq. (4) are set as follows

$$u^+(0) = 0, \quad c(0) = 0, \quad c'(0) = 1, \quad c''(0) = a \quad (5)$$

where a is a free (shooting) parameter.

It should be noted that model (4) is related to the Blasius equation in the particular case when $R_t = 1/2$, $a = 0$ and $x \ll 1$.

3 Constants of wall turbulence

Let us suppose that for $z^+ \rightarrow \infty$ the streamwise mean velocity profile has a logarithmic asymptotic, i.e.

$$du^+ / dz^+ \rightarrow 1/kz^+.$$

Surmising that $\lim_{x \rightarrow \infty} I(x, R_t) = I_0(R_t)$ we have from the first equation (4): $du^+ / dz^+ = I^+ e^{-I_0} / z^+$, and therefore $I^+ = e^{I_0} / k$. The last equation gives the continuous spectrum of the turbulent scales $I^+ = I^+(R_t)$. Used the dynamic roughness Reynolds number in the form $R_t = w_0^+ I_0^+$ the equation $I^+ = e^{I_0} / k$ can be rewritten as follows

$$w_0^+ / k = R_t \exp[-I_0(R_t)]$$

For an uniqueness of the mean velocity profile we can suppose that for $k = const$ the second turbulent velocity scale has a stable value at small variations of the parameter R_t , i.e. $dw_0^+ / dR_t = 0$. It gives $R_t = R_t^* \approx 1.2193$. The turbulent boundary layer fundamental scale can be defined as $I_0^+ = e^{I_0(R_t^*)} / k$. This quantity approximately equals to the experimental value $I_0^+ \approx 9.16$ for $k = 0.39$.

The function $1/I^+ = ke^{-I_0}$ can be considered as a spectral density. The inverse length scale versus the Reynolds number is shown in Figure 2a. This type of a spectral density is similar to the spectral density of the streamwise velocity fluctuations in the turbulent boundary layers. The function $w_0^+ = w_0^+(R_t)$ is shown in Figure 2b. This type of spectrum is similar to the spectral density of the transversal velocity pulsation [15]. Both functions represent the constructive model of the hydrodynamic chaos in this theory of turbulence.

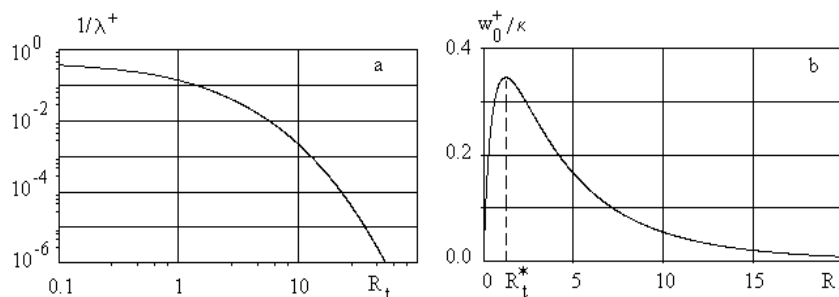


Figure 2: a) The inverse length scale $1/I^+$ versus the Reynolds number of dynamic roughness in double logarithmic scale. This type of a spectral density is similar to the spectral density of the streamwise velocity fluctuations in the turbulent boundary layers; b) the normalised turbulent velocity scale w_0^+/k versus the Reynolds number of dynamic roughness. This type of spectrum is similar to the spectral density of the normal to the wall velocity pulsation.

Our suggestion about the dynamical roughness structure in the wall region is that the parameter a fluctuates around the mean value $a = p/2$. This structure looks like furrows elongated along of the mean flow stream lines in the viscous sublayer (see, for instance, [16] where the visualisation of the coherent structure in the turbulent boundary layer is presented). In this case the first Eq. (4) can be integrated once and the standard logarithmic profile can be derived for $z^+ \gg I_0^+$:

$$u^+ = \frac{1}{k} \ln z^+ + c_0, \quad c_0 = \frac{1}{k} \int_0^\infty \frac{e^{I_0-I} - 1}{\sqrt{1+x^2}} dx - \frac{1}{k} \ln \frac{I_0^+}{2}.$$

Therefore, with the given constant k another constant of the mean velocity logarithmic profile can be calculated from the last equation.

4 Spectral characteristics of wall turbulence

The spectral density of the streamwise velocity pulsation can be defined as follows

$$\int F(k)dk = \int u^2(w)dw = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T u^2(t)dt = \langle u^2 \rangle$$

where w is the characteristic radian frequency, $k = w/U$ is the flow wave number (Taylor's frozen turbulence hypothesis). To compare the spectral density with experimental data let us suppose that $h_t \propto wh$, Therefore the Reynolds number calculated on the dynamical roughness parameters depends on the frequency,

$$R_t = wh^2 / \nu n^2 = w l_*^{+2} n / u_t^2 = k H I_*^{+2} u^+ / Re_*$$

where $n^2 = h_x^2 + h_y^2$, I_*^+ is the typical turbulent length scale of the streamwise velocity pulsation, H is the boundary layer height, $Re_* = Hu_t / \nu$. Due to this relationship the spectral characteristic of the turbulent flow is related to the eigen spectrum of the value problem (4-5). In general case it can be like the power series

$$F(k) = \sum_{i=1}^{\infty} c_i (I_*^+)^{-i}$$

Practically we can test one first term of this series. Then the spectral density can be proposed in the form

$$F(k) = c_K \frac{Hu_t^2}{I_*^+} = kc_K Hu_t^2 e^{-I_0[R_*(k)]} \tag{6}$$

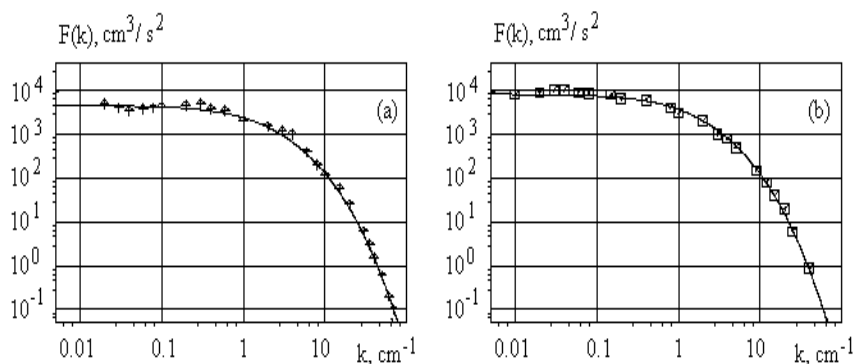


Figure 3: The estimated spectral density of the streamwise velocity pulsation in the turbulent channel flow (solid lines) for $z^+ = 4.9$ (a), and $z^+ = 11.7$ (b), and the experimental data [17]

where c_K is the normalising factor. Suggesting that the flow wave number depends on the dynamical roughness Reynolds number as the linear function, i.e. $k \propto R_t$, we have $c_K = 2.13 I_*^{+2} u^+ \langle u'^2 \rangle / u_t^2 Re_*$.

The spectral density given by Eq. 6 is shown in Figure 3 (solid lines) together with the experimental data [17] obtained in the turbulent channel flow. As it was established both spectral density parameters slowly depend on the distance from the wall. The best correlation was found in the viscose sublayer for $z^+ = 4.9$ $I_*^+ = 6.76$ $c_K = 1.01$ (Figure 3a) and for $z^+ = 11.7$, $I_*^+ = 5.45$, $c_K = 1.82$ (Figure 3b).

5 Thermal turbulent boundary layer

The model of turbulent flow (4) was developed to estimate the turbulent transport of heat and impurities in the boundary layer. The mean temperature gradient can be written as follows (see [10-11] for details)

$$\frac{dT^+}{dz^+} = \frac{\text{Pr} \exp[-0.5 P_t c'(x) \ln(1+x_1^2)]}{\sqrt{1+x_1^2}} \tag{7}$$

where $T^+ = (T_g - \tilde{T})/T_*$, T_g is the surface temperature, \tilde{T} is the turbulent flow temperature, $T_* = q_H / (r c_p u_t)$ is the turbulent scale of temperature, q_H is the heat flux from the rigid surface to the flow, c_p is the specific heat at constant pressure of the gas, $x_1 = z^+ / I_T^+$, $I_T^+ = (k_h \text{Pr})^{-1}$ is the main turbulent scale of the thermal layer, $P_t = w_0^+ I_T^+ \text{Pr}$ is the Peclet number.

The boundary conditions for equation (12) on a smooth wall and for the long distance from the wall are given by

$$\begin{aligned} z^+ = 0: \quad T^+ = 0; \quad dT^+ / dz^+ = \text{Pr} \\ z^+ \rightarrow \infty: \quad dT^+ / dz^+ \rightarrow 1/k_h z^+, \end{aligned} \tag{8}$$

where k_h is a constant which approximately equals to the Karman constant.

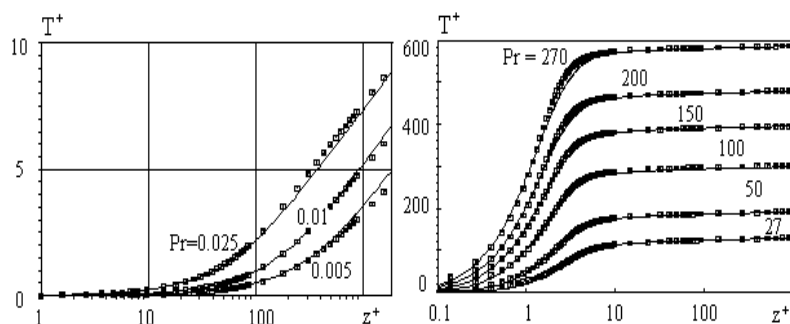


Figure 4. Mean temperature profiles in the turbulent boundary layer computed for the small (left) and for the large Prandtl numbers (right). The square symbols -the model of Sebeci [18]; the solid lines - the model (7-9).

In special case when $Pr \rightarrow 0$ a general solution of the boundary value problem (7)-(8) can be written as follows:

$$T^+ = \frac{1}{k_h} \text{Arsh}(k_h Pr z^+) \quad (9)$$

The mean temperature profiles computed on equations (7), (9) are shown in Figure 4 (right) by solid lines together with the profiles computed on the model of Sebeci [18] - the square symbols. For the best correlation the function $P_t(Pr)$ was numerically approximated as follows $P_t = -1.61 / (1 + 0.1 \ln Pr)$ in the range of the Prandtl number $2 < Pr < 10^3$. The agreement between two models in general is good.

It should be noted that equation (7) is a typical two-scale model describing the interaction between the transition layer with $l \approx l_0$ and thermal boundary layer with the main scale $l_T^+ = (k_h Pr)^{-1}$. Considering the mean flow as a "substance" with the Prandtl number $Pr = 1$ we can utilize equation (7) also to describe the mean velocity profile.

6 Multi-scale model

Analysing data shown in Figure 1a one can find the several layers of different scale:

- The viscous sublayer $z^+ \leq 1/k, A \approx z^+$;
- The transition layer which corresponds to the first maximum of A for $z^+ \approx I_0^+$;
- The logarithmic layer where $A \approx 1/k$;
- The mixing layer or the "wake region" [19] which corresponds to the second maximum of A for $z \approx H/2$;
- The free stream region where $A \rightarrow 0$ for $z \geq H$.

With this scales the model of turbulence (4) has been developed for the case of turbulent boundary layer flow in pressure gradient as follows

$$\frac{du^+}{dz^+} = \frac{e_0 \exp(I_0 - I)}{kI_0^+ \sqrt{1+x^2}} - \frac{I_0^+ p^+ e^{-I}}{k \sqrt{1+x^2}} \int_0^x \frac{e^I dx}{\sqrt{1+x^2}} - \frac{e_0}{kV_* \text{Re}_* \sqrt{1+\bar{z}^2}} + \frac{e(p^+) \sqrt{1+\bar{z}_0^2}}{kV_* \text{Re}_* (1+\bar{z}^2)} \text{ where}$$

$e_0 = 1 + (1 - e)/(kV_* \text{Re}_* \sqrt{1+\bar{z}_0^2})$, $V_* = 0.27$, $p^+ = \frac{n}{ru_*^3} \frac{\partial p}{\partial x}$ is the dimensionless pressure gradient, $\bar{z} = (z - z_0)/V_* H$, $z_0 = H/2$ is the middle position of the mixing layer.

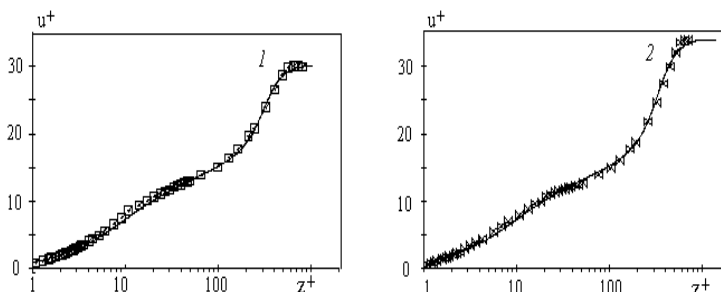


Figure 5: The estimated mean velocity profiles in the turbulent boundary layer in adverse pressure gradients computed for $p^+=0.023$; 0.0251 - the solid lines 1-2 accordingly, and experimental data (symbols) from the data base by Nagano *et al* [20].

The function of pressure gradient parameter has been estimated from the experimental data by Nagano *et al* [20] $e(p^+) \approx 118099p^{+3} - 2386.8p^{+2} + 36.579p^+ + 0.7881$ for adverse pressure gradients. The computed mean velocity profiles in the turbulent boundary layer in adverse pressure gradients are shown in Figure 5 together with the experimental data [20].

7 Final closure and conclusion

Generally speaking the Karman constant also should be defined as a function of this parameter because the two-scale model (7) depends on the length scale $I_v^+ = 1/k$ which can be related to the fundamental length scale I_0^+ . Used a symmetry of the multiple density of a probability distribution function $f_s(h, h_x, h_y, -h_t) = f_s(h, h_x, h_y, h_t)$ one can defined the length scale corresponding to a negative value $R_t = -R_t^*$ as follows $I_{in}^+ = I_n^+ \exp[I_0(-R_t^*)]$ and consequently the scale of velocity $w_{in}^+ = -R_t^* / I_{in}^+$. Both scales depend on the main scales w_0^+, I_0^+ due to a symmetry so that $I_{in}^+ = w_0^+, w_{in}^+ = -I_0^+$. Therefore the Karman constant should be defined as follows

$$k = \frac{1}{\sqrt{R_t^*}} \exp\left[\frac{I_0(R_t^*) + I_0(-R_t^*)}{2}\right] \quad (10)$$

The formula (10) gives $k = 0.3931$ for $R_t^* = 1.2193$. This result is presented in Figure 1a by the horizontal line 5.

We have demonstrated the completely closed theory of wall turbulence. The main parameter of this theory was established as the Reynolds number of dynamic roughness $R_t = w_0^+ I^+ = hh_t / (h_x^2 + h_y^2)$. This parameter has been calculated within a theory based on the mean velocity profile asymptotic behaviour as $R_t^* = 1.2193$. The Karman constant and the turbulent boundary layer fundamental length scale have been defined as functions of this parameter. The estimated parameters $k = 0.3931$, $I_0^+ \approx 9.16$ and mean velocity and temperature profiles are in a good agreement with the available experimental data.

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