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5.2.2. Математические, статистические и инструментальные методы экономики (физико-математические науки, экономические науки)

КОНСТРУИРОВАНИЕ ИНФОРМАЦИОННОЙ МЕРЫ УРОВНЯ СИСТЕМНОСТИ (ЭМЕРДЖЕНТНОЧТИ) СИСТЕМ С УЧЁТОМ ВЗАИМОСВЯЗИ БАЗОВЫХ ЭЛЕМЕНТОВ В ПОДСИСТЕМАХ

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В статье рассматривается обобщение классической формулы Хартли для измерения информации в системе с учётом не только её иерархической структуры, состоящей из подсистем различной сложности, как в предложенном Е.В.Луценко в 2002 году системном обобщении формулы Хартли, но и с учётом взаимосвязей между элементами. Ранее классическая исходная формула Хартли, описывающая количество информации в множестве из N элементов, была расширена для учёта подсистем системы, которые включают группы элементов на различных уровнях иерархии. В основе системного обобщения классической формулы Хартли 2002 года лежит гипотеза о существовании системной информации, т.е. о том, что информация содержится не только в множестве базовых элементов, но и в подсистемах различной сложности и различных уровней иерархии системы. Эта информация, содержащейся не в базовых элементах, а в подсистемах системы, названа системной информацией. Однако в ранее предложенном системном обобщении формулы Хартли не учитывались сила и знак взаимосвязей между базовыми элементами в подсистемах. В данной работе для оценки степени эмерджентности (уровня системности) системы предложена формула, соотносящая количество системной информации с количеством информации не только в множестве базовых элементов, но и в их подсистемах разных уровней иерархии и различной степени связности внутри подсистем. В работе представлено обобщённое выражение для вычисления информации в системе, которое включает учёт силы и знака взаимосвязей между элементами в подсистемах. Включение этих параметров позволяет более точно измерить

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5.2.2. Mathematical, statistical and instrumental methods of economics (physical and mathematical sciences, economic sciences)

DESIGNING AN INFORMATION MEASURE OF THE LEVEL OF SYSTEMICITY (EMERGENCE) OF SYSTEMS, TAKING INTO ACCOUNT THE RELATIONSHIP OF THE BASIC ELEMENTS IN THE SUBSYSTEMS

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The article considers a generalization of the classical Hartley formula for measuring information in a system taking into account not only its hierarchical structure consisting of subsystems of varying complexity, as in the systemic generalization of the Hartley formula proposed by E.V. Lutsenko in 2002, but also taking into account the interrelations between elements. Earlier, the classical original Hartley formula, which describes the amount of information in a set of N elements, was expanded to take into account the subsystems of the system, which include groups of elements at different levels of the hierarchy. The systemic generalization of the classical Hartley formula of 2002 is based on the hypothesis of the existence of systemic information, i.e. that information is contained not only in the set of basic elements, but also in subsystems of varying complexity and different levels of the system hierarchy. This information, contained not in the basic elements, but in the subsystems of the system, is called systemic information. However, the previously proposed systemic generalization of the Hartley formula did not take into account the strength and sign of the interrelations between the basic elements in the subsystems. In this paper, a formula is proposed for assessing the degree of emergence (systemicity level) of a system, which relates the amount of system information to the amount of information not only in a set of basic elements, but also in their subsystems of different levels of hierarchy and different degrees of connectivity within the subsystems. The paper presents a generalized expression for calculating information in a system, which includes taking into account the strength and sign of the relationships between elements in subsystems. Including these parameters allows for a more accurate measurement of information reflecting the interdependence of elements. The article also offers a информацию, отражающую взаимозависимость элементов. Статья также предлагает методику для расчёта этой информации с использованием матрицы взаимосвязей и выражений, которые учитывают количество подсистем и их взаимодействия. Кроме того в статье предлагается обобщение ранее предложенного автором (Е.В.Луценко, 2002) коэффициента эмерджентности, названного в честь Хартли, учитывающее не только количество подсистем различных уровней сложности на различных иерархических уровнях организации систем, но и силу и направление взаимосвязей между базовыми элементами в подсистемах. Кроме того рассматриваются следующие вопросы: зависимость количества информации в системе от числа иерархических уровней в ней при различных силах взаимосвязей между элементами, соблюдение принципа соответствия для предложенных в данной работе обобщённых выражений с ранее полученными, асимптотическое поведение выражения для количества информации в системе с учётом силы и знака взаимосвязей между базовыми элементами в подсистемах, а также численный пример. В результате получена улучшенная модель, учитывающая не только количество элементов, но и их связи, что способствует более точному измерению информации в сложных системах

Ключевые слова: ХАРТЛИ, ИНФОРМАЦИЯ, СИСТЕМА, ИЕРАРХИЯ, ПОДСИСТЕМА, ВЗАИМОСВЯЗЬ, ЭМЕРДЖЕНТНОСТЬ, УРОВЕНЬ СИСТЕМНОСТИ, ВЗАИМОЗАВИСИМОСТЬ, МАТРИЦА ВЗАИМОСВЯЗЕЙ

methodology for calculating this information using a matrix of relationships and expressions that take into account the number of subsystems and their interactions. In addition, the article offers a generalization of the emergence coefficient previously proposed by the author (E.V. Lutsenko, 2002), named after Hartley, which takes into account not only the number of subsystems of different levels of complexity at different hierarchical levels of system organization, but also the strength and direction of the relationships between basic elements in the subsystems. In addition, the following issues are considered: the dependence of the amount of information in the system on the number of hierarchical levels in it with different strengths of relationships between elements, compliance with the principle of correspondence for the generalized expressions proposed in this work with those obtained earlier, the asymptotic behavior of the expression for the amount of information in the system taking into account the strength and sign of the relationships between the basic elements in the subsystems, as well as a numerical example. As a result, an improved model is obtained that takes into account not only the number of elements, but also their relationships, which contributes to a more accurate measurement of information in complex systems

Keywords: HARTLEY, INFORMATION, SYSTEM, HIERARCHY, SUBSYSTEM, INTERACTION, EMERGENCE, LEVEL OF CONSISTENCY, INTERDEPENDENCE, MATRIX OF RELATIONSHIPS

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CONTENT

1. Introduction


Ralph Vinton Lyon Hartley, 30.11.1888 – 1.05.1970

In 1928, Ralph Vinton Lyon Hartley proposed the famous and now classical formula for the amount of information in a set consisting of N elements (1) [31]:

$$I = LogN \quad (1)$$

In a number of works from 1990-2002, summarized in the monograph [1], the author formulated a hypothesis about the existence of systemic information, suggesting that information is contained not only in a set of basic elements, but also in subsystems of varying complexity and different levels of system hierarchy.

Based on this hypothesis, a systemic generalization of the Hartley formula for the amount of system information in a system (2) consisting of W basic elements and W subsystems of different levels of hierarchy, including 2, 3,…, W elements, is proposed:

$$I = \log_2 \left(\sum_{m=1}^{W} C_W^m \right) \quad (2)$$

Where:

- is the number of ways to select elements from , i.e. this is the number of subsystems of the m-th level of the system hierarchy: namely, subsystems in which there are m elements.C_W^mmW

Also, in the work [1], a quantitative information measure of the level of systematicity (degree of emergence) of the system (3) is proposed, which is the ratio of the amount of system information in the system (2) to the amount of information in the set of basic elements of this system (1):

$$\Psi = \log_2 \left(\sum_{m=1}^{W} C_W^m \right) / \log_2 W \quad (3)$$

Since expression (3) is based on Hartley's classical expression for the amount of information {1} and on its systemic generalization (2), in [1] it was named by the author as the Hartley emergence coefficient in his honor. However, this name misled many authors, as they began to think that expression (3) was proposed by Hartley himself, whereas in fact it was proposed by the author in [1] and was simply named by the author in honor of Hartley [1-30] (Figure 1):

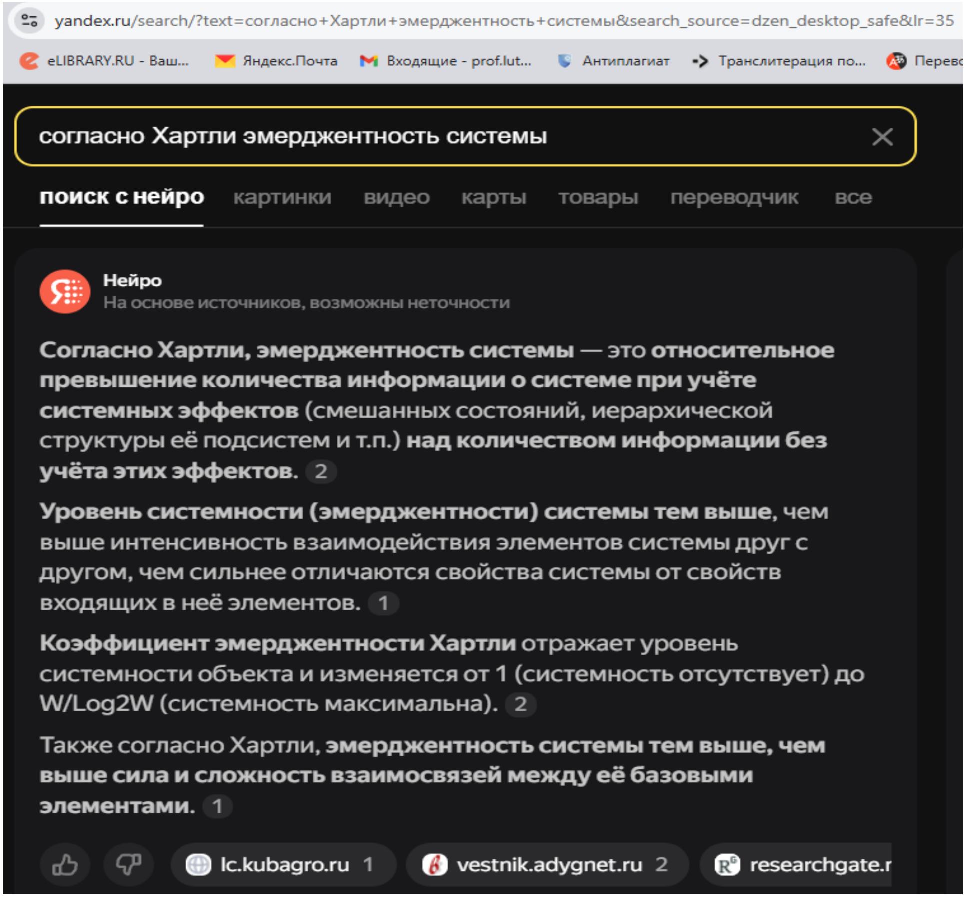

Figure 1. Query result: "according to Hartley, system emergence"

However, expressions (2) and (3) are somewhat simplified, since they do not take into account the strength and sign of the relationships between the basic elements in the subsystems. They only count the number of subsystems at different levels of the system hierarchy, and these subsystems are considered equally with the basic elements of the system in the same way as in the classical Hartley formula (1).

Meanwhile, according to the systemic information theory (SIT), proposed by the author in 2002 [1-30], the system asymptotically transforms into a set of basic elements with a decrease in the strength of the relationships between its basic elements [14, 15, 16], from which it follows that the level of systematicity (emergence) of the system, i.e. the degree of difference (of the properties) of the system from (the properties) of the set of its basic elements, increases with an increase in the number and strength of relationships between the elements of the system.

This means that formulas (2) and (3) must be generalized by taking into account not only the number of subsystems at different levels of the hierarchy, but also the level of systematicity of these subsystems, proportional to the strength of the interrelations between elements in the subsystems, as well as in the system as a whole.

2. Method

We will assume that the strength and sign of the relationship between the basic elements are determined by a matrix similar in structure to the similarity matrix, which has an external origin and reflects the specificity of the basic elements of the system and the structure of the system as a whole.

3. Results

3.1. Generalized expression for the amount of information in a system taking into account the strength and sign of the relationships between basic elements in subsystems

Since relationships can only be in pairs of basic elements, then:

1. We will assume that each basic element makes the same contribution to the overall strength of the subsystem connections as two maximally connected basic elements. Thus, the average strength of the connections of the basic elements is equal to 1.

2. A subsystem of m basic elements makes a contribution to the total strength of the system's connections equal to the average strength of the connections of all combinations of pairs of basic elements included in this subsystem (other options can be chosen, but in this paper we will focus on this).

3. The average strength of the connection of a subsystem of m basic elements is equal to the average strength of the connections of the basic elements, equal to 1, and pairs of basic elements.

Using the formula for the total strength of the connections of the m-th subsystem:

$$S_{total} = m + \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} S_{j,k},$$

and the expression for the average strength of the connection of the m-th subsystem:

$$\bar{S}_m = \frac{S_{total}}{m + C_m^2} = \frac{m + \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} S_{j,k}}{m + \frac{m(m-1)}{2}},$$

we can write a formula for the whole system:

$$I = \log_2 \left(\sum_{m=1}^{W} \sum_{i=1}^{C_W^m} \bar{S}_m \right),$$

Where:

$$\bar{S}_m = \frac{m + \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} S_{j,k}}{m + \frac{m(m-1)}{2}}.$$

Substituting this into , we obtain the final generalized expression for the amount of information in the system, taking into account the strength and sign of the relationships between the basic elements in the subsystems (4):I

$$I = \log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{C_W^m}\frac{m + \sum_{j=1}^{m-1}\sum_{k=j+1}^{m} S_{j,k}}{m + C_m^2}\right). \qquad (4)$$

Let's analyze this formula (4) step by step to reveal its meaning.

Formula:

$$I = \log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{C_W^m}\frac{m + \sum_{j=1}^{m-1}\sum_{k=j+1}^{m} S_{j,k}}{m + C_m^2}\right)$$

1. External sum by:m

- is the sum of all values from 1 to . Here can be the maximum number of elements or levels in some system.$\sum_{m=1}^{W}$ mWW

2. Internal sum by:i

- — this sum assumes that we are going through all possible combinations of , where is the number of combinations of -elements of . That is, for each we consider all possible combinations of .$\sum_{i=1}^{C_W^m}$ $iC_W^m C_W^m mWmi$

3. Numerator:

The numerator expresses a combination of two parts: - is simply the value of . - is the sum over all pairs of and , where . Each element represents some connection or interaction between elements of and within a set of dimension . The value may be associated with some measure or parameter of the interaction between these elements (for example, it may be the weight of an edge in a graph or a measure of the dependence between elements of a system). The sum over all pairs reflects all interactions between elements within the group under consideration.mm $\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} S_{j,k}$ $jk1 \leq j < k \leq mS_{j,k}jkmS_{j,k}(j,k)$

Thus, the numerator is the sum of the magnitude and the sum of all interactions between elements in a group of elements.mm

4. Denominator:

The denominator of the formula contains the expression: - — here the first term is added to the number of pairs that can be formed from the elements. The value is the number of combinations of 2 of the elements, that is, the number of all possible pairs of elements in the m-th group (subsystem). This number can also be interpreted as the number of all possible interactions between pairs of basic elements of the group (subsystem).$m + \frac{m(m-1)}{2}$ mm $\frac{m(m-1)}{2}$ mC_m^2

5. Logarithm:

$$\log_2(\ldots)$$

The logarithm to base 2 is used to calculate the "measure of information" or entropy in bits, which is related to the number of possible states or combinations that can occur in the system under consideration. The summation in the numerator and denominator, and the use of the logarithm at the end, indicate that this formula estimates the "amount of information" that is created by the interactions of the elements of the system.

Final explanation:

This formula generally estimates the information (or entropy) of a system of elements, where for each -subsystem the interactions between all elements within the group are considered. The formula takes into account the number of possible combinations of elements, their interactions (via), and uses a logarithm to calculate the measure of information. The denominator normalizes the sum by representing some form of "weights" or "scaling" for each combination.$WmS_{j,k}$

Thus, the meaning of this formula is to calculate information about the system taking into account the strength of internal relationships between all possible pairs of basic elements in subsystems and the dependence on the number of elements in these subsystems.

Let us rewrite expression (4) so that the number of combinations is expressed through variables and simplify the expression for greater convenience in carrying out numerical calculations.

Original formula:

$$I = \log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{C_W^m}\frac{m + \sum_{j=1}^{m-1}\sum_{k=j+1}^{m} S_{j,k}}{m + C_m^2}\right),$$

Where:

- — the number of combinations of by , is expressed as ,$C_W^m W m C_W^m = \binom{W}{m} = \frac{W!}{m!(W-m)!}$

- is the number of combinations of 2, expressed as .$C_m^2 m C_m^2 = \binom{m}{2} = \frac{m(m-1)}{2}$

Rewriting the formula

1. Replace with .$C_W^m \frac{W!}{m!(W-m)!}$

2. Replace with .$C_m^2 \frac{m(m-1)}{2}$

Now the formula will look like this:

$$I = \log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{\frac{W!}{m!(W-m)!}}\frac{m + \sum_{j=1}^{m-1}\sum_{k=j+1}^{m} S_{j,k}}{m + \frac{m(m-1)}{2}}\right).$$

Simplifying the denominator

The denominator can be simplified:

$$m + \frac{m(m-1)}{2} = \frac{2m + m(m-1)}{2} = \frac{m^2 + m}{2} = \frac{m(m+1)}{2}.$$

Thus, the formula becomes:

$$I = \log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{\frac{W!}{m!(W-m)!}}\frac{m+\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}S_{j,k}}{\frac{m(m+1)}{2}}\right).$$

The final expression for calculations (5):

$$I = \log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{\frac{W!}{m!(W-m)!}}\frac{2\left(m+\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}S_{j,k}\right)}{m(m+1)}\right) \quad (5)$$

Now expression (4) is written in terms of variables, where the number of combinations is expressed explicitly, which simplifies its use for calculations.

3.2. Research of the expression for the amount of information in the system taking into account the strength and sign of the relationships between the basic elements in subsystems

3.2.1. Dependence of the amount of information in the system on the number of hierarchical levels in it with different strengths of relationships between elements

We use the gamma function in expression (5) instead of the factorial, so that we can make a step of changing the indices under the sum less than 1 (6). To replace factorials with the gamma function, we can use the property:

$$n! = \Gamma(n+1)$$

Then the number of combinations can be expressed through the gamma function as follows:C_W^m

$$C_W^m = \frac{\Gamma(W+1)}{\Gamma(m+1)\cdot\Gamma(W-m+1)}$$

This will allow us to use non-integer values for the variables and . Let us rewrite expression (5) for using the gamma function:WmI

$$I = log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{\frac{\Gamma(W+1)}{\Gamma(m+1)\cdot\Gamma(W-m+1)}}\frac{2\left(m+\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}S_{j,k}\right)}{m(m+1)}\right) \quad (6)$$

The Python code below allows you to plot a 2D graph of the dependence of at (Figure 1):***I*** W $0 \le S_{j,k} \le 1$

```
import numpy as np
import matplotlib.pyplot as plt
from math import factorial, log2


# Функция для вычисления I
def calculate_I(W, Sjk=1):
    I_values = []
    for w in range(1, W + 1):
```

```
        sum_outer = 0
        for m in range(1, w + 1):
            # Вычисление числа сочетаний для m из W
            comb = factorial(w) // (factorial(m) * factorial(w - m))
            sum_inner = 0
            # Определяем количество пар (j, k) для каждого m
            pair_count = (m * (m - 1)) // 2  # Количество пар (j, k) для m
            # Если S_{jk} = 1, то сумма будет pair_count * S_{jk}, если
S_{jk} = 0, то 0
            sum_Sjk = pair_count * Sjk
            sum_inner += 2 ** (m + sum_Sjk)  # Суммирование значений
            sum_outer += sum_inner / (m * (m + 1))  # Добавление в общую
сумму

        # Если сумма больше 0, рассчитываем логарифм
        I = log2(sum_outer) if sum_outer > 0 else 0
        I_values.append(I)

    return I_values


# Задаем диапазон W = 20
W_max = 20

# Список значений Sjk от 0 до 1 с шагом 0.1
Sjk_values = np.arange(0, 1.1, 0.1)

# Построение графиков для разных значений Sjk
plt.figure(figsize=(12, 6))

for Sjk in Sjk_values:
    I_values = calculate_I(W_max, Sjk)
    plt.plot(range(1, W_max + 1), I_values, label=f'Sjk = {Sjk:.1f}')

plt.xlabel('W')
plt.ylabel('I')
plt.title('График зависимости I от W для разных значений Sjk (W = 20)')
plt.legend()
plt.grid(True)
plt.show()
```

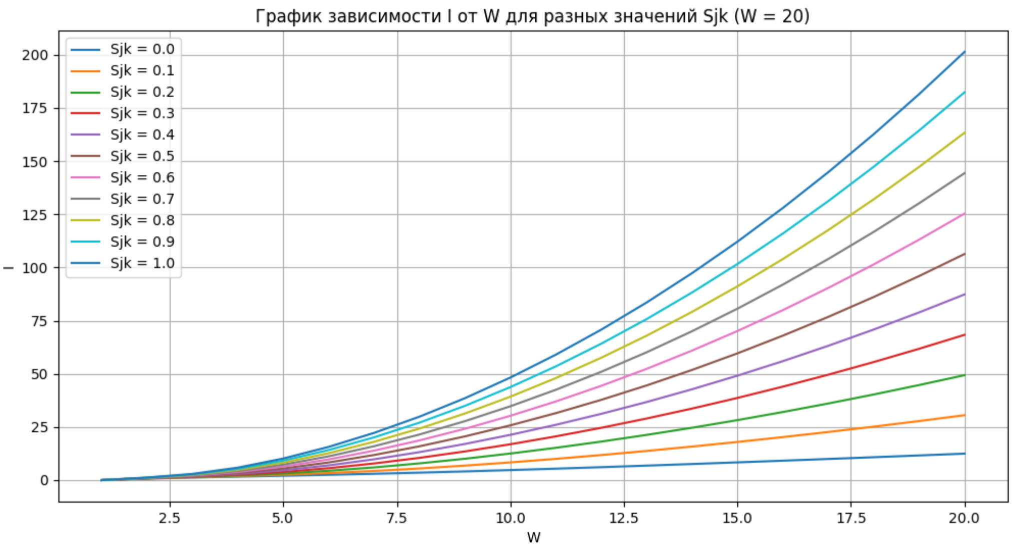

Figure 1. Graph of dependence on$IW 0 \leq S_{j,k} \leq 1$

3.2.2. Compliance with the principle of conformity

Let us check the fulfillment of the principle of correspondence for expression (4) with expression (2), of which it is a generalization, which is obligatory for more general theories. This means that when the strength of the relationship in pairs of basic elements of subsystems (4) tends to 1, expression (4) should asymptotically tend to (2).

The correspondence principle (or asymptotic approximation) suggests that a complex system reduces to a simpler one under certain conditions. In your case, the condition reduces to the fact that the element tends to 1, which means that all interactions between elements are considered equal. Consider how expression (4) will approach the simpler expression (2) under this condition.S_{ij}

Original expression:

$$I = \log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{C_W^m}\frac{m + \sum_{j=1}^{m-1}\sum_{k=j+1}^{m} S_{j,k}}{m + C_m^2}\right)$$

Condition :$S_{ij} \rightarrow 1$

When all elements tend to 1, then the sum over all pairs and in the numerator becomes simply the number of possible pairs, that is:$S_{ij}jkC_m^2$

$$\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} S_{j,k} \rightarrow \sum_{j=1}^{m-1}\sum_{k=j+1}^{m} 1 = \binom{m}{2} = \frac{m(m-1)}{2}.$$

Let's substitute this into the original expression:

Substituting , we get:$S_{ij} = 1$

$$I = \log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{C_W^m}\frac{m + \frac{m(m-1)}{2}}{m + \frac{m(m-1)}{2}}\right).$$

Let's simplify the expression:

In the numerator and denominator we have the same expressions that cancel each other out:

$$I = \log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{C_W^m} 1\right).$$

Now inside the sum over , since is simply the number of combinations of -elements from .i $\sum_{i=1}^{C_W^m} 1 = C_W^m mW$

So, the expression simplifies to:

$$I = \log_2\left(\sum_{m=1}^{W} C_W^m\right).$$

Conclusion:

We got the expression:

$$I = \log_2\left(\sum_{m=1}^{W} C_W^m\right),$$

which is what the original formula tends to for . This is the simpler expression that takes into account only the number of combinations for each , and it is asymptotically obtained from the original formula.$S_{ij} \rightarrow 1 C_W^m m$

This means that expression (4) is indeed a generalization of the systemic generalization of Hartley's formula (2), taking into account the strength of the relationships between the elements of the subsystems.

3.2.3. Asymptotic behavior of the expression for the amount of information in the system taking into account the strength and sign of the relationships between the basic elements in subsystems

To analyze the asymptotic behavior of expression (4) with unlimited increase in , we consider two of its cases: when for all and when .$W S_{jk} = 1 j,k S_{jk} = 0$

1. Original expression:

$$I = \log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{C_W^m}\frac{m + \sum_{j=1}^{m-1}\sum_{k=j+1}^{m} S_{jk}}{m + C_m^2}\right)$$

Where:

- — the maximum value of the index,$W m$
- — binomial coefficient,$C_W^m \binom{W}{m}$
- are the elements of the matrix that are either 1 or 0.$S_{jk} S$

2. Case 1: for everyone$S_{jk} = 1 j,k$

If , then the sum in the numerator takes on a value equal to the number of pairs for each , which is . Thus, the expression for the numerator becomes:$S_{jk} = 1 \sum_{j=1}^{m-1}\sum_{k=j+1}^{m} S_{jk}\, j,k\, m \binom{m}{2} = \frac{m(m-1)}{2}$

$$m + \frac{m(m-1)}{2} = m + \frac{m^2 - m}{2}.$$

Now let's substitute this into the original formula:

$$I = \log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{C_W^m}\frac{m + \frac{m^2 - m}{2}}{m + C_m^2}\right).$$

For large and , the quadratic and linear terms in the numerator and denominator dominate. For simplicity, assume that for large we have:$m W m$

$$\frac{m + \frac{m^2 - m}{2}}{m + C_m^2} \sim \frac{\frac{m^2}{2}}{m} = \frac{m}{2}.$$

Now let's estimate the sum by:m

$$\sum_{m=1}^{W}\sum_{i=1}^{C_W^m}\frac{m}{2}\sim\sum_{m=1}^{W}\binom{W}{m}\frac{m}{2}.$$

When the sum over will dominate due to the high order of the binomial coefficients, and the result will behave asymptotically as . Thus, the expression for will have the asymptotics:$W\rightarrow\infty mW^2I$

$$I\sim\log_2(W^2)=2\log_2W.$$

3. Case 2: for everyone$S_{jk}=0j,k$

If , then the sum in the numerator is reduced to:$S_{jk}=0$

$$m+\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}0=m.$$

Thus, the expression for the numerator is simply equal to , and then for each we get:mm

$$I=\log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{C_W^m}\frac{m}{m+C_m^2}\right).$$

When the linear term in the numerator dominates. When the value is large compared to the sum tends to 1, and thus will tend asymptotically to:$W\rightarrow\infty C_m^2mI$

$$I\sim\log_2(W).$$

4. Result:

- For all , the asymptotic behavior of expression (4) will be .$S_{jk}=1j,kI\sim 2\log_2W$
- For all , the asymptotic behavior of expression (4) will be .$S_{jk}=0j,kI\sim\log_2W$

3.2.4. Numerical example

The numerical example is calculated based on expressions (2) and (5)

$I=\log_2\left(\sum_{m=1}^{W}C_W^m\right)$	(2)
$I=\log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{\frac{W!}{m!(W-m)!}}\frac{2\left(m+\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}S_{j,k}\right)}{m(m+1)}\right)$	(5)

using the Python program below according to all numbered expressions for W=5.

The matrix of the strength of the relationships between the basic elements is filled with random numbers:

```
import itertools
import math
import random


# Функция для вычисления количества сочетаний C(n, m)
def combinations(n, m):
    if m == 0 or m == n:
        return 1
    return math.comb(n, m)


# Функция для создания матрицы взаимодействий
def calculate_interaction_matrix(numbers):
    n = len(numbers)
    interaction_matrix = [[0] * n for _ in range(n)]

    # Заполняем симметричную матрицу случайными числами
    for i in range(n):
        for j in range(i + 1, n):
            interaction_matrix[i][j] = random.uniform(0.01, 0.99)  #
Случайное значение в диапазоне от 0.01 до 0.99
            interaction_matrix[j][i] = interaction_matrix[i][j]  #
Симметричность матрицы

    # На диагонали все единицы
    for i in range(n):
        interaction_matrix[i][i] = 1.0

    return interaction_matrix


# Функция для нормализации матрицы взаимодействий в диапазон [0, 1]
def normalize_interaction_matrix(interaction_matrix):
    flat_values = [interaction_matrix[i][j] for i in
range(len(interaction_matrix)) for j in
                   range(i, len(interaction_matrix))]
    min_value = min(flat_values)
    max_value = max(flat_values)
    # Применяем нормализацию
    for i in range(len(interaction_matrix)):
        for j in range(i, len(interaction_matrix)):
            normalized_value = (interaction_matrix[i][j] - min_value) / (
                        max_value - min_value) if max_value != min_value
else 0
            interaction_matrix[i][j] = normalized_value
            interaction_matrix[j][i] = normalized_value  # Симметричная
матрица
    return interaction_matrix


# Число базовых элементов
W = 5
numbers = list(range(1, W + 1))  # Список чисел от 1 до W

# Матрица взаимодействий
interaction_matrix = calculate_interaction_matrix(numbers)
normalized_interaction_matrix =
normalize_interaction_matrix(interaction_matrix)
```

```
# Формула 2: I = log2(∑_(m=1)^W ∑_(i=1)^(C_W^m) (m + ∑_(j=1)^(m-1)
∑_(k=j+1)^m S_(j,k)) / (m + C_m^2))

# Рассчитаем для разных вариантов S_(j,k):
def calculate_I_for_S(jk_values):
    total_sum = 0
    for m in range(1, W + 1):
        # Количество всех подсистем размера m
        num_subsystems = combinations(W, m)
        # Сумма для всех подсистем размера m
        for _ in range(num_subsystems):
            # Общая сила связей, используя матрицу взаимодействий для
заданных значений S_(j,k)
            num_pairs = m * (m - 1) // 2
            sum_S_jk = 0
            for i in range(m):
                for j in range(i + 1, m):
                    sum_S_jk += jk_values[i][j]  # Используем заданное
значение S_(j,k)
            S_total = m + sum_S_jk
            # Средняя сила связей
            avg_S = S_total / (m + num_pairs)
            total_sum += avg_S
    return math.log2(total_sum)


# Варианты для S_(j,k):
S_zero = [[0] * W for _ in range(W)]  # Все S_(j,k) = 0
S_one = [[1] * W for _ in range(W)]  # Все S_(j,k) = 1
S_random = [[random.uniform(0.01, 0.99) for _ in range(W)] for _ in
range(W)]  # 0 < S_(j,k) < 1, случайные числа

# Расчеты для различных вариантов
I_2_zero = calculate_I_for_S(S_zero)
I_2_random = calculate_I_for_S(S_random)
I_2_one = calculate_I_for_S(S_one)

# Формула 1: I = log2(∑_(m=1)^W C_W^m)
sum_C = sum(combinations(W, m) for m in range(1, W + 1))
I_1 = math.log2(sum_C)
print(f"I для формулы 1 (без учета связей): \t{I_1:.4f}")
print(f"I для формулы 2 (все S_(j,k) = 0 ): \t{I_2_zero:.4f}")
print(f"I для формулы 2 (0 < S_(j,k) < 1 ): \t{I_2_random:.4f}")
print(f"I для формулы 2 (все S_(j,k) = 1 ): \t{I_2_one:.4f}")

# Вывод нормализованной матрицы взаимодействий
print("\nМатрица взаимосвязей базовых элементов:")
for row in normalized_interaction_matrix:
    print([f"{value:.4f}" for value in row])

# Подсистемы различных уровней иерархии
print("\nПодсистемы:")
for level in range(1, W+1):  # Уровни с 1 по 5
    subsystems = list(itertools.combinations(range(1, W + 1), level))
    print(f"\nПодсистемы {level}-го уровня иерархии:")
    for i, subsystem in enumerate(subsystems, 1):
        print(f"Подсистема {i}: {subsystem}")
```

Below are the results of this program.

I for formula 1 (excluding connections): 4.9542

I for formula 2 (all S_(j,k) = 0): 4.2479
I for formula 2 (0 < S_(j,k) < 1): 4.7405
I for formula 2 (all S_(j,k) = 1): 4.9542

Matrix of relationships of basic elements:
['1.0000', '0.6941', '0.6946', '0.1886', '0.0000']
['0.6941', '1.0000', '0.2750', '0.3465', '0.0230']
['0.6946', '0.2750', '1.0000', '0.2341', '0.8109']
['0.1886', '0.3465', '0.2341', '1.0000', '0.7724']
['0.0000', '0.0230', '0.8109', '0.7724', '1.0000']

Subsystems of the 1st level of hierarchy:
Subsystem 1: (1,)
Subsystem 2: (2,)
Subsystem 3: (3,)
Subsystem 4: (4,)
Subsystem 5: (5,)
Subsystems of the 2nd level of hierarchy:
Subsystem 1: (1, 2)
Subsystem 2: (1, 3)
Subsystem 3: (1, 4)
Subsystem 4: (1, 5)
Subsystem 5: (2, 3)
Subsystem 6: (2, 4)
Subsystem 7: (2, 5)
Subsystem 8: (3, 4)
Subsystem 9: (3, 5)
Subsystem 10: (4, 5)
Subsystems of the 3rd level of hierarchy:
Subsystem 1: (1, 2, 3)
Subsystem 2: (1, 2, 4)
Subsystem 3: (1, 2, 5)
Subsystem 4: (1, 3, 4)
Subsystem 5: (1, 3, 5)
Subsystem 6: (1, 4, 5)
Subsystem 7: (2, 3, 4)
Subsystem 8: (2, 3, 5)
Subsystem 9: (2, 4, 5)
Subsystem 10: (3, 4, 5)
Subsystems of the 4th level of hierarchy:
Subsystem 1: (1, 2, 3, 4)
Subsystem 2: (1, 2, 3, 5)
Subsystem 3: (1, 2, 4, 5)

Subsystem 4: (1, 3, 4, 5)
Subsystem 5: (2, 3, 4, 5)
Subsystems of the 5th level of hierarchy:
Subsystem 1: (1, 2, 3, 4, 5)

3.3. Generalized expression for the Hartley emergence coefficient taking into account the strength and sign of the relationships between the basic elements in subsystems

Also, in the work [1], a quantitative information measure of the level of systematicity (degree of emergence) of the system (3) is proposed, which is the ratio of the amount of system information in the system (2) to the amount of information in the set of basic elements of this system (1):

$$\Psi = \log_2\left(\sum_{m=1}^{W} C_W^m\right)/\log_2 W \tag{3}$$

Let us generalize the expression for the Hartley emergence coefficient (3) taking into account the relationships between the basic elements in the subsystems (5):

$$\Psi = \log_2\left(\sum_{m=1}^{W}\sum_{i=1}^{C_W^m}\frac{m+\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} S_{j,k}}{m+C_m^2}\right)/\log_2 W \tag{8}$$

4. Discussion

The generalized formula for calculating information in a system (expression (4)) proposed in this paper represents a significant expansion of not only the classical Hartley formula (1), but also its previously proposed (E.V. Lutsenko, 2002) systemic generalization (2), since it takes into account not only the number of subsystems in the hierarchy, but also the strength of the relationships between the basic elements in the subsystems.

It is important to emphasize that based on this model, the system is considered as a multi-layer structure, where each level of the hierarchy and each subsystem can have different characteristics of the relationships between the elements.

Unlike classical Hartley information theory, which focuses exclusively on the discreteness of elements and their possible states, the generalized formula integrates additional parameters, such as the strength and sign of the relationships between elements, which allows for more accurate modeling of real systems. Within this model, the system is not reduced to a simple set of discrete elements, but is a structure with interdependent parts, where the strength of the connections significantly affects the overall level of information.

However, despite the complexity that the generalized model faces, it can be used to more accurately describe not only natural but also artificial systems, where the relationships between system components may be heterogeneous, change over time, or depend on external factors. This opens up new opportunities for research in the field of system information theory, especially in the context of analyzing network structures, biological systems, or economic models, where the interaction of elements has an important impact on the dynamics of the system.

In addition, the generalized formula (4) is amenable to numerical analysis using software tools, as shown in the example with Python code, which allows us to practically study the dependence of the information level on various factors, such as the number of elements in the system (W) and the strength of the relationships between them. The constructed graphs of the dependence of information on the number of hierarchy levels confirm that an increase in the number of elements and the complexity of subsystems contributes to an increase in the information level in the system. This, in turn, leads to a more complete understanding of the processes of emergence in complex systems.

However, it is worth noting that the proposed model does not yet take into account many possible factors, such as the dynamics of changes in relationships over time, the influence of external factors, or more complex types of interactions between elements. These aspects can be included in future extensions of the model, which will allow obtaining more accurate results in specific areas, for example, in the analysis of complex network structures or in modeling interactions in biological systems.

The correspondence principle tested within the framework of the work also confirms the correctness of the proposed model. When the strength of the relationships tends to 1, the model really approaches the systemic generalization of the classical Hartley formula (2), which is an important aspect of the theoretical correctness of the generalized theory. In this sense, the proposed work has significant value for further research, since it harmoniously combines the classical theory with new, more complex ideas about systems and their information characteristics.

Thus, the generalized expression for calculating information in a system proposed in this paper (expression (4)) represents a significant expansion of both the classical Hartley formula (1) and its systemic generalization proposed by E.V. Lutsenko in 2002 (expression (2)). This expansion is important because the new formula takes into account not only the number of subsystems in the hierarchical structure of the system, but also the strength of the relationships between the basic elements that make up these subsystems. Thus, the proposed model allows for a more accurate assessment of the information characteristics of complex systems, which is an important step toward taking into account the structural and dynamic aspects of the interaction of system elements.

5. Conclusions, limitations and prospects

In this paper, a generalized expression for calculating the amount of information in a system was proposed, taking into account not only the number of elements and subsystems, but also the strength of the relationships between the elements of these subsystems. Generalization of Hartley's expressions based on the system theory of information, as well as the introduction of the concept of emergence and the proposed formula for calculating the strength of connections between basic elements allow us to more accurately describe information processes in complex systems.

However, despite significant achievements, the proposed model has a number of limitations. In particular, at this stage it is impossible to take into account all possible types of relationships between system elements, including more complex forms of dependencies that may appear in real systems. In addition, the proposed methodology requires improvement in terms of computational algorithms, since the use of multiple sums and combinations makes calculations very resource-intensive, especially with an increase in the size of the system.

Prospects for further research include:

1. Refinement and expansion of the model: It is important to take into account various types of relationships between elements, including nonlinear and complex dependencies, which can lead to more accurate estimates of the level of systematicity and emergence in real systems.

2. Developing more efficient computational methods: The use of numerical modeling methods, such as graph-based and optimization methods, will significantly speed up calculations and reduce resource consumption.

3. Application in various fields: The proposed model can be used to analyze information in such fields as systems theory, ecology, economics, as well as in the development of more accurate and adaptive models in the field of artificial intelligence and machine learning, where it is important to take into account the relationships and interactions between the components of the system.

Overall, the generalization of the Hartley model taking into account the relationships between system elements opens up new horizons for the analysis and prediction of the behavior of complex systems and can become an important tool in information and systems theory.

Желающие ознакомиться с данной работой на русском языке могут сделать это по ссылке: https://www.researchgate.net/publication/385788762

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